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Markets liquidity risk under extremal dependence: Analysis with VaRs methods

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ABSTRACT

Value-at-Risk (VaR) is a widely used tool for assessing financial market risk. In practice, the estimation of liquidity extreme risk by VaR generally uses models assuming independence of bid–ask spreads. However, bid–ask spreads tend to occur in clusters with time dependency, particularly during crisis period. Our paper attempts to fill this gap by studying the impact of negligence of dependency in liquidity extreme risk assessment of Tunisian stock market. The main methods which take into account returns dependency to assess market risk is Time series–Extreme Value Theory combination. Therefore we compare VaRs estimated under independency (Variance–Covariance Approach, Historical Simulation and the VaR adjusted to extreme values) relatively to the VaR when dependence is considered. The efficiency of those methods was tested and compared using the backtesting tests. The results confirm the adequacy of the recent extensions of liquidity risk in the VaR estimation. Therefore, we prove a performance improvement of VaR estimates under the assumption of dependency across a significant reduction of the estimation error, particularly with AR (1)-GARCH (1,1)-GPD model.

1. Introduction

During the last decade, stock markets of emerging countries registered a financial instability. Indeed, at the end of 2007, a depreciation of emerging equities recorded by the fall of the MSCI[1] index by 9%. The EMBI[2] spread widened by 48 basis points following a drastic fall of bond prices, and some emerging currencies depreciated strongly (Bulletin of the Bank of France, 2007). Subsequently, the overall liquidity of all markets exhibits a downward trend.

The stock market liquidity is a critical element in investment decision in foreign stock exchange. Much of this interest comes from liquidity problems linked to the absorption of orders, without resulting a significant movement of prices and within a relatively short time (Black, 1971; Harris, 1990; Kyle, 1985). This problem recorded, especially in emerging markets and considerably during periods of shocks and stock market crashes.

According to Aubier and Le Saout (2002), liquidity risk corresponds to the loss from the cost of liquidating a position. This risk increases during financial crises and results in the inability of the market to absorb order flow without provoking violent price adjustments. Typically, the market illiquidity appears as an important transaction costs, a low “turnover”, a low number of transactions or orders placed during the sessions, a higher market efficiency coefficient or even a higher bid–ask spread. However, as liquidity problems correspond to the compensation of cost, not the remuneration of risk (Amihud and Mendelson, 1986), the liquidity cost can be considered as an additional transaction cost (explicit costs related to expenses incurred, including the costs of processing orders, asset trading costs, storage costs and taxes associated to transactions – Glosten and Harris (1988) – and implicit costs). Because the bid–ask spread can approximate these costs, it is considered then as the most reliable indicator to reflect the financial markets liquidity. This factor means that investors who wish to lead a position will have to pay significant costs for doing so: they may incur considerable transaction costs, an waiting time relatively long due to the absence of counterpart or sell quickly at an unfavorable price.

During periods of financial stress, the illiquidity problem of financial markets is usually the result of problem of expectations coordination and of investors behavior which are closely related to the future prices anticipation (Masson, 1999). The fear that prices fall leads sales to the single direction that cause a drop in real price. At this time, and reinforced by mimetic behavior of uninformed participants, all investors wish to sell at the same time stripped the order book side of the buyer. The financial institutions, which are the “natural” contributors of liquidity, withdraw from the market or refrain from buying. There is then an excess supply of securities against a demand which tends to be reduced sharply following the mistrust vis-à-vis securities. It is the case where there is not any more investment strategy: Everybody wants to liquidate his position.

The bid–ask spread tends to increase significantly. It can therefore produce a panic equilibrium preceded by an extreme price volatility and leads to drying up of global liquidity with the disappearance of

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[1] MSCI: Morgan Stanley Capital International.
[2] EMBI: Emerging Markets Bond Index.

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any activity on the secondary market. It is clear that most markets have sometimes experience liquidity problems. Even markets that are highly liquid most of the time, their liquidity may "drop" occasionally during crises. The stock markets liquidity risk is therefore a financial risk potentially significant, long ignored by financial theory, which requires special tools for modelling and managing.

Referring to the Basel II and as a market risk, modeling the liquidity risk of financial markets can use measures like Value-at-Risk (Almgren and Chriss, 2001; Bertsimas and Lo, 1998; Hisata and Yamai, 2000; Shamroukh, 2000). This new measure is a risk indicator. It is used primarily for measuring aggregate risk because it allows to regroup the overall risk in a common unit of measure for all risks, whatever their nature (exchange rate, stocks, bonds, etc…). It expresses the loss from adverse movements of market prices. Adopted to liquidity risk, this new measure aims to consider the risk that must be managed during the liquidation of the portfolio. This new technique is heavily used in recent years by risk managers, in particular after the failures of some big U.S. banks in the early 90s. Several financial institutions, operating internationally, began to adopt Value-at-Risk, to manage, quantify and establish correct information about the risk of their portfolios. Inspired of this new position, Lawrence and Robinson (1998) propose a generic model that determines the maximum loss suffered by an investor, at a given level of probability, when it's remaining positions and ensure the liquidation of its portfolio at optimal speed. Besides, Häberle and Persson (2000) define a Value-at-Risk adjusted to liquidity as the potential loss upon conventional liquidation of a portfolio.

Two main methods for estimating VaR, proposed in the literature, are the parametric and nonparametric method. The famous parametric method is the Analytical VaR assuming normality of the statistical series. Whereas, Historical Simulation based on past changes of risk factors and Monte Carlo simulation based on a random generation process, stand for the most reliable nonparametric techniques.

However, during financial crisis periods, the expected loss on liquidation a position may be much higher than that achieved in normal period. Recently, Aubier (2006) points that extreme movements in French stock market liquidity may be crucial with respect to frequency and amplitude. He shows that the tails of the liquidity proxies distribution are thick. This result emphasizes the importance of extreme values that exist in the tails of distributions. The extreme liquidity situation is considered when the proxy's value exceeds some deterministic threshold. These extrema can not be adequately described by the Gaussian distribution. Therefore, it is necessary to readjust the classical models of VaR that appear ill-suited to the apprehension of extreme events related to liquidity crises. It would be useful to produce a risk measure of extreme liquidity. In this case, the extreme value theory can provide a solution through the VaR adjusted to extremes bid–ask spread of market (Aubier, 2007; Bangia et al., 1999).

To examine the behavior of extreme bid–ask spread of shares traded on emerging stock market, we have referred to many empirical studies which show that investment returns know an asymmetrical dependence: they are more dependent in crisis period than in calm period. Thus, a bit risky action – very liquid – usually, can be much less safe in times of crisis because of increased dependence of assets. The risk modeling assuming the dependence of financial series complements the range of measures VaR implicitly based on the market factors independence.

In our setting, research on extreme situations of Tunisian stock market are mainly interested in stock performance (Ghorbel and Trabelsi, 2008), in market index (Snoussi and El-Aroui, 2006) and in exchange risk (Ben Rejeb et al., 2012). No studies have assessed the market liquidity risk by the use of VaR measures under the hypothesis of the series dependence. This paper is the first to address this issue. Its aim is to evaluate liquidity risk of Tunisian stock market according to the various methods suggested (under dependent and independent series) and to compare their performance through backtesting.

The remainder of this paper is organized as follows. Section 2 advances the extreme value theory. Section 3 presents the methods of estimating VaRs dependent and independent. Section 4 reports the backtesting technique and tests associated with. Section 5 analyzes the empirical results and we give some concluded remarks in Section 6.

2. Extreme value theory

Extreme value theory (EVT) is a branch of the theory of order statistics that is specifically used to describe the rare event. It concerns the asymptotic distribution of the maximum or minimum standard from a series of *i.i.d* random variables designated by R_{t1}, R_{t2}, …, R_{tn} with a common distribution function F. It is then to determine the distribution to which M_n converges for larger values of n with $M_n = \max\{R_{t1}, \ldots, R_{tn}\}$. The extreme value theory specifies the form of this distribution. There are two main branches of this theory. Generalized Extreme Value distribution "GEV" developed by Fisher and Tippett (1928) and Generalized Pareto Distribution "GPD" developed by Belkema and de Haan (1974) and Pickands (1975). The theorem of Fisher and Tippett (1928) focuses on the asymptotic distribution of maximum standardized by block (block monthly, quarterly, semiannual, etc..). However, for a given normalized constants $a_n > 0$ and $b_n \in R$, Fisher and Tippett (1928) show that:

$$\lim_{n\to+\infty} \Pr\left(\frac{M_n - b_n}{a_n} \le x\right) = \lim_{n\to+\infty} F^n(a_n x + b_n) = G_\zeta(x). \text{ for all } x \in R_+. \quad (1)$$

Where: a_n is interpreted as a scale measure and b_n is a location measure. The Fisher–Tippet theorem states that if the standardized maximum (1) converges to some non-degenerate distribution function, it must be a Generalized Extreme Value (GEV) distribution of the form:

$$\begin{cases} G_\zeta(x) = exp(-(1+\zeta x)^{\frac{-1}{\zeta}}); \text{if} \quad \zeta \neq 0 \text{ and } (1+\zeta x) > 0 \\ G_\zeta(x) = exp(-exp(-x)); \text{if} \quad \zeta = 0 \quad for\, all\ x \ge 0. \end{cases} \quad (2)$$

Where $G_\zeta(x)$ is named the Generalized Extreme Value distribution (GEV) and ζ the shape parameter. ζ is also called tail index because it characterizes the extreme behavior of distribution function (if $\zeta = 0$ Gumbell distribution (thin tail), if $\zeta > 0$ Frêchet distribution (fat tail) and if $\zeta < 0$ Weibull distribution (without tail)).

Modeling only block maxima data is inefficient if other data on extreme values are available. A more efficient alternative approach that utilizes more data is to model the behavior of extreme values above some high threshold. This method is often called Peaks Over Thresholds (POT). Developed by Belkema and de Haan (1974) and Pickands (1975), this second technique showed that for a high threshold μ, a Generalized Pareto Distribution (GPD) of extrema with distribution function $G_{\zeta,\sigma}$, is presented as follows:

$$\begin{cases} G_{\zeta,\sigma}(y) = 1 - \left(1 + \zeta\left(\frac{y}{\sigma}\right)\right)^{\frac{-1}{\zeta}}; \quad \text{if} \quad \zeta \neq 0 \\ G_{\zeta,\sigma}(y) = 1 - \exp\left(-\frac{y}{\sigma}\right); \qquad \text{if} \quad \zeta = 0 \end{cases} \text{for all } y \in R_+^*. \quad (3)$$

Generalized Pareto Distribution (GPD) is characterized only by scale and shape parameters, σ and ζ (the location parameter is null). $y_i = R_{ti} - \mu$ is the exceedance over the threshold. There are many methods to estimate those parameters like Hill estimator, Maximum Likelihood estimator (QML) or Probability Weighted Moment estimator (PWM). In this paper we use the estimators based on PWM, proposed by Hosking and Wallis (1987), considered more