On the effects of changing mortality patterns on investment, labour and consumption under uncertainty

Christian-Oliver Ewald\textsuperscript{a,}\textsuperscript{*}, Aihua Zhang\textsuperscript{b}

\textsuperscript{a} Adam Smith Business School - Economics, University of Glasgow, Glasgow, G12 8QQ, United Kingdom
\textsuperscript{b} University of Leicester, Department of Mathematics, Leicester, LE1 7RH, United Kingdom

**HIGHLIGHTS**

- We study consumption, labour and portfolio decisions in presence of mortality risk.
- Use UK actuarial life tables spanning the time period from 1951–2060.
- Historical changes in mortality cause significant changes in the agent's decisions.
- Changes contribute up to 5% to GDP growth during the period from 1980 until 2010.

**ARTICLE INFO**

Article history:
Received September 2016
Received in revised form January 2017
Accepted 28 January 2017
Available online 8 February 2017

**JEL classification:**
G11
J11
J22
C61

**Keywords:**
Life-cycles
Portfolio investment
Flexible labour
Age-dependent mortality rates
Uncertain lifetime

**ABSTRACT**

In this paper we extend the consumption–investment life cycle model for an uncertain-lived agent, proposed by Richard (1974), to allow for flexible labour supply. We further study the consumption, labour supply and portfolio decisions of an agent facing age-dependent mortality risk, as presented by UK actuarial life tables spanning the time period from 1951–2060 (including mortality forecasts). We find that historical changes in mortality produce significant changes in portfolio investment (more risk taking), labour (decrease of hours) and consumption level (shift to higher level) contributing up to 5% to GDP growth during the period from 1980 until 2010.

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**1. Introduction**

Lifetime consumption and investment models for infinitely lived agents have been considered by various authors, including Merton (1969, 1971), Bodie et al. (1992) as well as Bodie et al. (2004). The setup in all of these contributions is very similar, they all study the problem of maximizing expected discounted utility under consideration of a utility function which includes consumption and in some cases leisure, over the life time of a representative agent. Bodie et al. (1992) considered an exogenously given retirement age and left it as an open question, to determine the optimal retirement age within an optimal stopping context. This problem has now been considered by Dybvig and Liu (2010). Zhang (2010) considered retirement age as exogenously given, but allowed for fully flexible labour supply, in essence including retirement as an option for the agent. Davis et al. (2006) argued that equity holdings over the life cycle in the classical Merton (1969, 1971) models were unrealistically high and emphasize the aspect of borrowing costs/constraints. They did not account for flexible labour and utility from leisure however.

In reality of course agents are not infinitely lived. Richard (1974) extended Merton's (1971) model to allow for a finitely lived agent with a random time of death. He introduced a bequest motive and life insurance into the model and considered the problem of optimal investment into the life insurance product. Using
the Hamilton–Jacobi–Bellman framework he derived analytic expressions for the optimal portfolio rule, consumption rate and life insurance under constant relative risk aversion (CRRA). In that case, mortality enters into the optimal portfolio rule, which is the same fraction as in Merton (1971) but multiplied by the ratio of total wealth (including mortality dependent human wealth, i.e. future incomes until death) to financial wealth. However, Richard (1974) did not consequently study how changes in mortality affected his optimal portfolio rule. Optimal consumption in Richard (1974) is a time dependent fraction of total wealth, but from the expression derived, it is not clear how consumption shifts and how consumption growth is affected as a consequence of the changes in mortality. Richard (1974) did not allow for flexible labour decisions either.

Milevsky and Young (2007) modified the framework presented by Richard (1974) to take account of some institutional issues related to the purchase and payout of annuities. In fact their focus was on the optimal annuitization when the agent is already retired and does not receive any labour income. Milevsky and Young (2007) did account for mortality, but by using a Hamilton–Jacobi–Bellman approach, real mortality data do not enter their model directly, but through a suitably parametrized Comperetz–Makeham hazard rate function. In addition, the resulting Hamilton–Jacobi–Bellman equation has been linearized, leading to approximate solutions only. A semi-analytic solution was presented for the case of constant force of mortality only. Huang et al. (2012) study a (Yaari, 1965) et al framework with stochastic force of mortality but focus on consumption only, leaving out stochastic investment and labour. In conclusion, neither Richard (1974) nor Milevsky and Young (2007) or Huang et al. (2012) did study the effects that historical changes in mortality rates cause on the agent’s optimal strategy.

Pang and Warshawsky (2010) also studied a portfolio problem, involving risk-less and risky assets as well as annuities. Agents in their model have exposure to mortality risk as well as uninsured health care costs. Their model was in discrete time and no attempt was made to solve the model analytically. Instead the model was solved numerically and results are based on simulation. Their main observation is that health spending risk drives the agent’s portfolio to shift from risky assets to safer assets. As in Richard (1974) and Milevsky and Young (2007), their study does not involve an investigation on how changes in actually observed and predicted mortality rates affect the agent’s optimal strategy.

More recently, Gahramanov and Tang (2013) have presented a paper in which they considered the retirement problem in a continuous time model with time varying mortality. However their work differs from ours, in that they focused on the retirement problem with mortality being given by an explicit analytic function as in Feigenbaum (2008). Furthermore, they did not allow for investment into risky assets.

One of the main contributions of this article is the inclusion of time varying, general mortality risk into a continuous time stochastic life time consumption model, where a representative agent chooses consumption, labour supply and portfolio investment into a risk-less and a risky asset and in consequence a rigorous study on how historically observed changes in the mortality patterns affect the agent’s decisions of portfolio selection, consumption and labour supply. We adopt a CRRA type of utility function measuring utility from consumption against dis-utility from supplying labour. We assume no bequest motive, and in consequence the agent’s optimal life insurance strategy is to contract their respective wealth to be transferred to a life insurance company at the time of their death in exchange for a fairly priced annuity as proposed by Yaari (1964, 1965) and Blanchard (1985).1 To solve our model, rather than using the Hamilton–Jacobi–Bellman framework, which seems less flexible in the context of general time varying mortality curves, we use a combination of Martingale techniques that have evolved from the Mathematical Finance literature, see for example the exposition in Korn and Korn (2001) and Zhang (2008) or the original work by Pliska (1986), Karatzas et al. (1987) and Cox and Huang (1989).2 The use of these methods enables us to derive analytic expressions for the optimal consumption, labour supply and portfolio investment process in the presence of mortality risk. We are further able to derive a compact form for the Euler equation of consumption growth. As a first result we find that the effect of mortality risk on consumption and labour supply is through the Lagrange multiplier of the associated static constrained optimization problem only, and as such it shifts consumption and labour supply, but has no effect on the Euler equation. This effect was not observed in Richard (1974). Mortality risk also affects optimal portfolio investment, but in a more subtle way than in Richard (1974) due to the presence of flexible labour.

Generally, the presence of mortality in a lifetime consumption context leads to a number of interesting effects and trade-offs, which also have existing models have not been able to capture and quantify. Longer life expectancy will emphasize the aspect of precautionary savings for old age. In addition fear of death might encourage people to consume their goods sooner than later (while still alive). Both of these mechanisms cause an effect where an increase in mortality increases current consumption. However, when life expectancy increases, longer (working) lives will increase human wealth and thus increase current consumption and investment. This mechanism works in the opposite direction, i.e. an increase in mortality contributes to a decrease in current consumption. Finally, risk taking behaviour in investment will also be altered, as long-term investment horizons will increase in length and thus making risky assets more attractive.

We also derive a closed-form expression for the elasticity of consumption with respect to the mortality rate. Using realistic parameters we find that this elasticity is negative, within the range of 0 (i.e. zero mortality rate) to −0.53 (equivalent to a mortality rate of 0.002 which corresponds roughly to the mortality rate of a 39 year old UK male). In the empirical part of the paper we have used actual and forecasted mortality curves as obtained from statistical life tables supplied by the UK’s Office for National Statistics covering the years from 1951 until 2060. Substituting these curves into our model we observe that keeping all other parameters constant, changes in the mortality curves from 1980 to 2010 lead to a shift in consumption upwards of roughly 5%, contributing to a total of approximately 100% in real GDP growth in the UK from 1980 to 2010.3 We also observe that optimal labour supply in effect of the same changes of the mortality curves is reduced by 4%, from about 40.2 to 38.7 h per week from 1980 to 2010. Finally, portfolio investment into the risky asset is increased by a factor of roughly 6%, financing the reduction in labour and increase in consumption.

We conclude that historical changes in mortality risk do indeed have significant impact on consumption spending, labour supply and portfolio investment.

The remainder of the paper is organized as follows. In Section 2, we set up our model and derive some basic equations, while in

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1 Yaari (1964, 1965) and Blanchard (1985) did not consider risky investment and (Blanchard, 1985) only considered a constant mortality rate.

2 By considering the mortality rates obtained from the Office for National Statistics as deterministic piecewise linear functions, it is possible to solve the model via the Hamilton–Jacobi–Bellman equation. This requires to solve the corresponding PDEs on 110 intervals, each according to one year between 1951 and 2060 and gluing the solutions together at the respective boundaries.

3 Historical data for real GDP have been obtained via https://docs.google.com/spreadsheets/cid=0AonYz5z4MzIz+GrO43OzGzT1EWkVpx1X1VW86LTd1U3cg#/gid=1. The shift in consumption over the whole data period from 1951 until 2060 is about 12%.
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