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Parameter estimation in commodity markets: A filtering approach

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Abstract

The application of Kalman filtering methods and maximum likelihood parameter estimation to models of commodity prices and futures prices has been considered by several authors. The usual method of finding the maximum likelihood parameter estimates (MLEs) is to numerically maximize the likelihood function. We present, as an alternative to numerical maximization of the likelihood, a filter-based implementation of the expectation maximization (EM) algorithm that can be used to find the MLEs. Finite-dimensional filters are derived that allow the MLEs of a commodity price model to be estimated from futures price data using the EM algorithm without calculating Kalman smoother estimates.

JEL classification: C13; G13

Keywords: Filtering; Parameter estimation; Futures price; EM algorithm

1. Introduction

The purpose of this paper is to provide a demonstration of the main steps of a filtering approach to parameter estimation for a model of commodity spot prices

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(such as crude oil) using futures price data. A one-factor model for the spot commodity price is used to illustrate the approach. This is done to avoid drawing the focus away from the new filters and the overall approach which are our main contribution. However, the methodology can be used for multi-factor models with some modifications. The characteristic shared by many of the continuous-time futures, forward, and bond price models discussed in the finance literature is that under a log-transformation and once appropriately discretized their state-space representations belong to the class of affine Gaussian state-space models. We shall consider only commodity price models and futures prices. The commodity price models of Gibson and Schwartz (1990), Schwartz (1997), Schwartz (1998), Schwartz and Smith (2000), and Manoliu and Tompaidis (2002) are examples with different parameterizations but an affine Gaussian form.

Complicating the implementation of commodity market models is that one or more of the factors may be unobservable. In practice meaningful spot prices for some commodities are not available. What is often reported as the 'price' of an asset is the futures price of the contract closest to maturity. In a model with two factors, such as the spot price and the convenience yield, both may be unobservable while what is observed is the term structure of futures prices for contracts with a quantity of the physical asset underlying. Both the problems of calibration and estimation of unobservable quantities fit naturally into the framework of filtering. The Kalman filter has been applied to both of these problems by Schwartz (1997), Schwartz and Smith (2000), and Manoliu and Tompaidis (2002) for various multi-factor models to both calibrate the model parameters to market data and to estimate the time series of the unobservable factors. The method of estimating model parameters in all of these previous studies was the direct approach where the likelihood function itself was computed and maximized numerically.

Rather than computing the maximum likelihood parameter estimates (MLE) by direct maximization of the likelihood function the expectation maximization (EM) algorithm can be used. The EM algorithm is a general iterative algorithm for maximizing the likelihood. Each iteration consist of two steps: expectation (E-step) and maximization (M-step). Following the techniques of Elliott and Krishnamurthy (1999) we derive explicit finite-dimensional filters necessary to obtain maximum likelihood estimates of the model parameters via the EM algorithm. The filters derived in this paper allow the E-step to be done without calculating Kalman smoother estimates normally used to implement the EM algorithm. As in Elliott and Krishnamurthy (1999) the filter-based approach to the EM algorithm used in this paper has certain advantages over the standard smoother-based implementation of the EM algorithm. For example, since only a forward pass through the data is required the filter-based approach will be at least twice as fast as a smoother-based approach which requires a forward-backward pass scheme. Further, the EM algorithm is well suited to parallel implementation on a multi-processor computer system (see, Elliott and Krishnamurthy, 1999). Other, possibly model specific, computational advantages may be realized using a filter-based implementation of the EM algorithm rather than a smoother-based implementation. For example, for the model considered in this paper and other constant coefficient models, the steady