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Computing the market price of volatility risk in the energy commodity markets

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ABSTRACT

In this paper, we demonstrate the need for a negative market price of volatility risk to recover the difference between Black–Scholes [Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–654]/Black [Black, F., 1976. Studies of stock price volatility changes. In: Proceedings of the 1976 Meetings of the Business and Economics Statistics Section, American Statistical Association, pp. 177–181] implied volatility and realized-term volatility. Initially, using quasi-Monte Carlo simulation, we demonstrate numerically that a negative market price of volatility risk is the key risk premium in explaining the disparity between risk-neutral and statistical volatility in *both* equity and commodity-energy markets. This is robust to multiple specifications that also incorporate jumps. Next, using futures and options data from natural gas, heating oil and crude oil contracts over a 10 year period, we estimate the volatility risk premium and demonstrate that the premium is negative and significant for all three commodities. Additionally, there appear distinct seasonality patterns for natural gas and heating oil, where winter/withdrawal months have higher volatility risk premiums. Computing such a negative market price of volatility risk highlights the importance of volatility risk in understanding priced volatility in these financial markets.

1. Introduction

The importance of energy markets has increased with the development of futures and options markets, and with the always-important impact of energy on the economy. In this paper, we seek an in-depth understanding of priced volatility in the energy markets, as well as quantitatively displaying the mirror-image aspect of the energy and equity markets: Whereas prices and volatilities are negatively correlated in the equity markets, they tend to display a *positive* correlation in the energy markets. Because of this positive correlation, computing a negative market price of volatility risk in energy markets may imply a negative market price of *commodity risk* in the energy markets and consequently an upward-bias of energy futures contracts prices relative to expected spot prices.

Financial research has made numerous advances in testing the sensitivity of a given data-generating process to changes in the instantaneous parameters that govern it. Currently there are a wealth of parametric models attempting to explain stock price movement. The most notable extensions to the Black and Scholes (1973) model are the inclusion of stochastic volatility à la Heston (1993), and the inclusion of jumps by Bates (1996) and others. Recent additions such as volatility jumps introduced by Duffie et al. (2000) and Bates' crash risk are further advancements to the well-tested Black–Scholes model. A problem for current researchers is the ability to reconcile time-series and cross-sectional differences in spot and option prices by fitting a given underlying model to capture the distributions of both returns.

The parameter of particular interest is the market price of volatility risk. While extensive research has focused on stochastic volatility models, there is conflicting information on the impact of the market price of volatility risk. Recently, findings in Bakshi and Kapadia (2003), Coval and Shumway (2001), Pan (2002), Doran (2007) address the direction and magnitude of the market price of volatility risk, but with contradictory conclusions. Our hypothesis is that the market price of volatility risk is indelibly linked to the bias in Black–Scholes implied volatility. If the market price of volatility risk is significant and negative, this potentially explains the upward-bias observed in Black–Scholes implied volatility (henceforth, BSIV) as well as contributing to the Bates' (1996) finding that out-of-the money (OTM) puts are expensive relative to other options (the so-called "volatility skew"). Additionally, Eraker (2004) and Bates (2000) have documented that selling options results in Sharpe ratios that are significantly higher than the Sharpe ratio of traditional equity portfolios. These results suggest large premiums for exposure to volatility risk, and lend justification for option traders tendency to be short options.

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Another strand of the literature has focused on the appropriate econometrics to estimate continuous-time models. Chernov and Ghysels (2000) use the Gallant and Tauchen (1998) EMM technique, Pan (2002) applied an IS-GMM framework, and Jones (2003) and Eraker (2004) have used Bayesian analysis to arrive at their estimates. These works have improved our understanding of equity market pricing dynamics and the risk premium within these markets by combining spot and options data on the S&P index. By comparison, the work in energy markets incorporating options is mostly unexplored. As Broadie et al. (2007) point out, it is very difficult to arrive at precise parameter estimates for multiple risk premia, especially when one is the volatility risk premium. Noting this issue, we attempt to estimate the volatility risk premium in energy markets by combining both implied and realized volatility in two-step estimation procedure. Using Black (1976) implied volatility (BIV) and implementing the relationship between expected volatility and instantaneous volatility as given in Aït Sahalia (1996), we infer the instantaneous risk-neutral parameter estimates from the discrete-time analogue. The market price of volatility risk is then deduced via calibration from these instantaneous parameters and the difference between expected realized volatility and the actual realized volatility whilst avoiding the under-identification problem.

The findings suggest a significant negative market price of volatility risk for three energy commodities – natural gas, crude oil and heating oil. Additionally, there appears to be a strong seasonal component to the volatility risk premium for natural gas and mild seasonality in heating oil. For robustness, in conjunctions with the volatility risk premium, the market price of risk and the correlation between the price and volatility innovations are estimated in the Hansen (1982) GMM framework, in a test for model mis-specification and stability of the parameter estimates. While we are unable to make a definitive statement on the commodity price risk premium, the volatility risk premium remained negative and significant using this alternative specification.

The remainder of the paper is organized as follows. Section 2 will review the current findings on the market price of volatility risk. Section 3 will introduce the simulation and the findings for the various parametric-models tested. Section 4 will detail the estimation procedure and its application to the market price of volatility risk. Section 5 concludes.

2. The volatility risk premium

The current state of the literature on volatility risk premium has focused on equity markets, in part due to the prevalence of option data on indices such as the S&P 500. Much of the attention paid to energy markets focuses on modeling and estimating convenience yields.[1] The general findings suggest that convenience yields are positive, since futures prices are generally below spot, are negatively correlated with inventories, and tend to be time-varying. While estimation of the convenience yield is typically conducted using spot and futures prices, estimating the market price of volatility risk needs be derived from information in *option prices*. With increased access to energy option data, estimation of the volatility risk premium is now more reliable. By using the realized volatility of the futures contract price, and implied volatility from the options on those futures, the volatility risk premium can be inferred.

For equities, the consensus of works such as Bakshi and Kapadia (2003), Carr and Liuren (2004), Coval and Shumway (2001) find that the market price of volatility risk is negative. Several authors, such as Pan (2002) and more recently, Broadie et al. (2007), disagree with the marginal impact of this risk parameter, noting that empirically disentangling multiple risk premiums is problematic. Our intuition regarding the sign of the market price of volatility risk is informed in part by the notion that options are purchased as hedges against significant declines in the equity market, and buyers of the options are willing to pay a premium for such downside protection.[2] In addition to the high Sharpe ratios in trading option, pointed out by Bates (2000) and Eraker (2004); Jackwerth and Rubinstein (1996) have also suggested that at-the-money (ATM) implied volatilities are systematically higher than realized volatilities, a phenomenon that clearly be explained by a negative volatility risk premium.[3]

While implied volatilities are higher than realized volatilities in energy markets, too, the dynamics of the energy markets differ from those of equity markets in specific ways:

1. Higher market prices tend to coincide with higher volatility.
2. Beta coefficients tend to be negative for commodities markets.
3. There is a significant term-structure of volatility and (at least in some markets) seasonality for energy prices.

In the presence of these major differences we show that the market price of volatility risk for energy contracts is also negative and significant. This is consistent with the Doran (2007) finding that option prices incorporate volatility risk in natural gas. First, simulation evidence will highlight that only the market price of volatility risk can explain the difference between implied and realized volatility, even in the presence of jumps. Second, we will estimate the volatility risk premium in three important energy commodities by incorporating the volatility difference between implied and realized and thus confirming the simulated numerical evidence.

3. Monte Carlo simulation

To demonstrate the need for a negative market price of volatility risk in energy, we adopt a quasi-Monte Carlo simulation to test several parametric-model candidates. The choice of possible model candidates is almost boundless, ranging from Gaussian, non-Gaussian, continuous and discrete – and including the Duffie et al. (2000) double-jump model. For the sake of brevity we will focus on the most common, i.e., Bates' (1996) stochastic volatility with jumps.[4] Our purpose here is to demonstrate that the volatility risk premium is negative and significant, and is the only parameter that can explain volatility differences between implied and realized volatility, even in the presence of jumps and jump premium.

We have chosen to model both the jump size risk as well as the jump intensity risk as proportional to the level of volatility, analogous to what is typically done in equities. To test for this proportional relationship in energy markets an ordered Probit test is conducted on the frequency of the jumps relative to volatility using crude oil futures. The independent variable chosen is beginning-of-month volatility, which is then sampled over the next 22 days for jump frequency and jump size. Daily futures price movements of 3% and 5% are selected to signify a jump in a given futures contract. If a month had one jump of 3% or 5%, the dependent variable is set equal to one; if there are two jumps the dependent variable is set equal to two, and so forth. The results of the ordered probit regression reveal positive *t*-statistics of 3.47 and 5.23 for absolute value

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[1] In particular, see Schwartz (1997), Hilliard and Reis (1998), Schwartz and Smith (2000), Routledge et al. (2000), Casassus and Collin-Dufresne (2005) and many others.

[2] This would have the appearance of buying market volatility, since high volatility coincides with falling market prices, as pointed out by French et al. (1987) and Nelson (1991).

[3] Jackwerth and Rubinstein (1996) demonstrate this by recovering the probability distributions from option prices.

[4] For a good discussion on option pricing model performance, see Bakshi et al. (1997) and Christoffersen et al. (2006).