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Almost periodic solutions for an impulsive delay model of price fluctuations in commodity markets

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ABSTRACT

In the present paper, we shall consider the following impulsive delay system for modeling the price fluctuations in single-commodity markets:

$$\begin{cases} \dot{p}(t) = F(p(t), p(t-h))p(t), \quad t \neq \tau_k, \\ p(t) = \varphi_0(t), \quad t \in [t_0 - h, t_0], \\ \Delta p(t) = I_k(p(t)), \quad t = \tau_k, \ k \in \mathbb{Z}. \end{cases}$$

Sufficient conditions are established for the existence of almost periodic solutions for this system. Piecewise continuous functions of the Lyapunov type as well as the Razumikhin technique have been utilized to prove our main results.

1. Introduction

Impulsive delay differential equations have conspicuously occupied a great part of researchers' interests for well over the last three decades. Indeed, it has been recently recognized that these equations do only generalize the corresponding theory of impulsive differential equations but also provide better mathematical descriptions for many real life applications. The publications [1–15] are devoted to the theory and applications of impulsive differential equations with or without delay.

Specifically, the dynamics of the economy is one of most actively developing research areas that can be represented by using impulsive delay differential equations of a certain type [16–18]. In particular, it has been noticed that these equations can provide adequate visualization for modeling the process of price fluctuations in single-commodity markets. Early authors often attributed these fluctuations to random factors such as weather change for agricultural commodities [19–21]. Other authors, however, speculated that fluctuations might be caused by dynamical characteristics of unstable economic systems [22–25]. Apart from some diversities in the authors' beliefs regarding this discussion, their work and that of others has played a fundamental role in the development of theory of nonlinear dynamics [26–31].

Searching the literature, one can realize that there has been intensive work regarding the study of periodic impulsive dynamical systems with or without delay; see for instance the Refs. [32–37] in which the existence of periodic solutions has been the main concern of the authors. Although it is known to be a natural generalization to the periodicity, the notion of almost periodicity has rarely been considered. The reader can easily figure out that a few results exist in this direction [38–42].

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The purpose of this paper is to study the almost periodic behavior of solutions for the impulsive delay model for price fluctuations in commodity markets. Piecewise continuous functions of the Lyapunov type as well as the Razumikhin technique have been utilized to prove the main results.

2. Problem statement; essential notation, definitions and lemmas

For a single market commodity, there are three effective variables: the quantity demanded D, and the quantity supplied S and its price p. It has been realized that there exist definite relationships among these three variables. These relationships, which are called the demand curve and the supply curve, are occasionally modeled by a demand function $D = D(p)$ and a supply function $S = S(p)$; both are dependent on the price variable p. If the rate of price changes with respect to time is assumed to be proportional to D and S, then the formulation of the following system proves helpful:

$$\frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}t} = f(D, S). \tag{1}$$

In particular, one can easily deduce that for linear demand and supply functions, system (1) has a dynamically stable solution of exponential type. Indeed, let $D = a - bp$ and $S = -c + \mathrm{d}p$ be given where $a, b, c, d \in \mathbb{R}^+ = [0, \infty)$. If f has the form $f = \alpha(D - S)$, $\alpha > 0$ where $D - S$ is the excess demand, then system (1) becomes

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \alpha(a + c - p(b + d)) = p\alpha\left(\frac{a + c}{p} - (b + d)\right). \tag{2}$$

The complementary and particular solutions of (2) can be easily attained.

In [43], Belair and Mackey considered Eq. (1) in order to study the dynamics of price, production and consumption for a particular commodity with price dependent delays. In [44], Mackey developed a price adjustment model for a single-commodity market with state dependent production and storage delays. Conditions for the equilibrium price to be stable are derived in terms of a variety of economic parameters. A particular case of the general model was considered by Farahani and Grove in [45]. Indeed, they proposed a system of the form

$$\frac{p'(t)}{p(t)} = \frac{a}{b + p^n(t)} - \frac{cp^m(t - h)}{d + p^m(t - h)}, \quad t \geq 0, \tag{3}$$

where $a, b, c, d, m, h \in \mathbb{R}^+$ and $n \geq 1$. Under the initial condition $p(t) = \varphi(t)$, $t \in [-h, 0]$, the authors established necessary and sufficient conditions for the existence of positive solutions of system (3). They proved, moreover, that these solutions oscillate to the unique positive equilibrium solution of (3). For the case where $t - h = g(t)$, system (3) was studied in [46]. Indeed, the author proved the existence of a unique positive bounded solution of (3). A more general system of the form

$$\begin{cases} \dot{p}(t) = F\big(p(t), p(t - h)\big)p(t), & t \in \mathbb{R},\ h \in \mathbb{R}^+ \\ p(t) = \varphi(t), & t \in [-h, 0], \end{cases} \tag{4}$$

was investigated by Rus and Iancu in [47]. The authors proved the existence and uniqueness of the equilibrium solution of the system and established some relations between this solution and the coincidence points.

If at certain moments in time the price is subject to short-term perturbations, then it is natural to expect the existence of an "irregular" solution for system (4). Indeed, the solution must have some jumps and these jumps will follow a specific pattern. Recently, it has been realized that impulsive delay differential equations provide the most adequate description for these phenomena. These equations have been widely used in many fields such as physics, chemistry, biology, population dynamics and industrial robotics as a control. The abrupt changes in the prices which may be caused by the environment, the competitive market or the government, can affect the transient behavior of the market. The result of this paper suggests that such a policy would be highly destabilizing and would either destabilize a previously stable market situation or exacerbate the instability of a market through an increase in the amplitude and period of oscillations in commodity prices.

Let $\mathbb{R}$ be the one-dimensional Euclidean space with the norm $|\cdot|$, $B_\nu = \{x \in \mathbb{R} : |x| \leq \nu\}$, $\nu > 0$ and suppose that $\Lambda \subset B_\nu$ where $\Lambda \neq \emptyset$. Let $\mathbb{B} = \big\{\{\tau_k\} : \tau_k \in \mathbb{R},\ \tau_k < \tau_{k+1},\ k \in \mathbb{Z}\big\}$ be the set of all sequences unbounded and strictly increasing with distance $\rho(\{\tau_k^{(1)}\}, \{\tau_k^{(2)}\})$ where $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$. Define the spaces $PC[\mathbb{R}, \mathbb{R}] = \big\{\varphi : \mathbb{R} \to \mathbb{R},\ \varphi$ is a piecewise continuous function with points of discontinuity at the first kind τ_k, $\{\tau_k\} \in \mathbb{B}$ at which $\varphi(\tau_k - 0)$ and $\varphi(\tau_k + 0)$ exist and $\varphi(\tau_k - 0) = \varphi(\tau_k)\big\}$ and $PC^1[\mathbb{R}, \mathbb{R}] = \big\{\varphi : \mathbb{R} \to \mathbb{R},\ \varphi$ is a function that is continuously differentiable everywhere except at the points τ_k, $\{\tau_k\} \in \mathbb{B}$ at which $\dot{\varphi}(\tau_k - 0)$ and $\dot{\varphi}(\tau_k + 0)$ exist, and $\dot{\varphi}(\tau_k - 0) = \dot{\varphi}(\tau_k)\big\}$.

Suppose that $\varphi_0 \in PC[\mathbb{R}, \Lambda]$ and $\|\varphi_0\| = \sup_{t\in\mathbb{R}} |\varphi_0(t)|$. We shall consider the following impulsive delay model for price fluctuations in commodity markets:

$$\begin{cases} \dot{p}(t) = F\big(p(t), p(t - h)\big)p(t), & t \neq \tau_k, \\ p(t) = \varphi_0(t), & t \in [t_0 - h, t_0], \\ \Delta p(t) = I_k(p(t)), & t = \tau_k,\ k \in \mathbb{Z}, \end{cases} \tag{5}$$

where: