Research Paper

Reliability-based design of tunnelling problems and insights for Eurocode 7

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ABSTRACT

The partial factor design approach has been suggested to replace the factor of safety design in geotechnical practice, such as the Eurocode 7 (EC7) for European countries and the load and resistance factor design (LRFD) for the North America. However, these design codes cover little about rock engineering principles and rock engineers struggle with the application of the partial factor design to rock engineering problems. This paper presents how reliability-based design (RBD) can provide insights to and help the evolution of the partial factor design approach for tunnelling problems. Compared with other reliability methods, the first-order reliability method (FORM) is consistent for different but mathematically equivalent limit state functions. The intuitive expanding ellipsoid perspective and the constrained optimization method for FORM help overcome the conceptual and computational barriers for practitioners. Three case studies are presented to show that RBD via FORM can determine the role (resistance or load factor) of input parameters on a case-by-case basis in ways that prescribed partial factors cannot, including a symmetrical roof wedge above a tunnel, a lined circular tunnel under non-hydrostatic in situ stresses and a circular tunnel reinforced by rockbolts considering multiple failure modes.

1. Introduction

The factor of safety design approach has long been dominating the geotechnical engineering although it cannot reflect how safe a geotechnical structure really is. This approach attempts to ensure that the loadings on a structure do not exceed the allowable limit and is, in some textbooks, called the allowable stress design (ASD), e.g. Baecher and Christian [1]. The uncertainties associated with material properties and in situ conditions are considered implicitly by a single factor of safety. A better alternative to ASD is the limit state design (LSD) which is based on predictions about how the design performs near failure, for example, the Eurocode 7 (EC7) [2] for the European countries and the load and resistance factor design (LRFD) [3] for the North America. Both codes use the characteristic values factored by partial factors to check the limit states. The difference lies in that EC7 applies the partial factors to individual parameters whereas LRFD to the combined resistance and load. With the development of the probabilistic analysis, reliability-based design (RBD) has become popular recently due to the fact that it is robust and the uncertainties are treated explicitly. Most commonly used reliability analysis methods include the first-order second-moment method (FOSM), point estimate method (PEM), first-order reliability method (FORM), second-order reliability method (SORM) and Monte Carlo simulation (MCS).

The partial factor design has been successfully applied to geotechnical problems worldwide. However, all the specifications in such design codes are pertaining to and calibrated by soil engineering problems. Rock engineering principles seem to be neglected in EC7 as pointed out by Harrison [4]. This may be due to historical reasons that less emphasis has been placed on the rock engineering during the preparation of the EC7 draft and due to the fact that the discontinuous, heterogeneous and anisotropic nature of the rock mass requires a great deal of empiricism be involved [5]. A committee on the evolution of EC7 has been formed to develop EC7 with regard to the rock engineering design. On the other hand, Low and Phoon [6] illustrated that RBD can play a complementary role to the partial factor design using some soil engineering problems. In this paper, some tunnelling problems are used to show that RBD, more specifically FORM-based design, is helpful in the evolution of EC7 for rock engineering. First, the difference between the partial factor design approach and RBD is illustrated by a circular tunnel in a Mohr-Coulomb ground. Then, different reliability analysis approaches (FOSM, PEM, FORM and SORM) are compared using a symmetrical roof wedge problem. Next, three case studies are employed to show the insights from RBD compared with the partial factor design. Finally, how RBD can complement the partial factor design for tunnelling problems is summarized.

2. Eurocode 7 design and FORM-based design

2.1. Eurocode 7 design

For the partial factor design, the design should satisfy the following condition.
\[
\sum \frac{R_{Ci}}{\psi_{Ri}} \geq \sum \psi_{Si} \Phi(1)
\]

where \( \psi_R \) is the resistance factor; \( R_C \) is the characteristic value of the resistance; \( \psi_S \) is the load factor; \( \Phi(1) \) is the characteristic value of the load.

The characteristic value of the resistance and load should be selected, based on Clause 2.4.5.2(2)\( \text{P} \) in EC7, as 'a cautious estimate of the value affecting the occurrence of the limit state' and can be selected by the statistical method if the distribution of this parameter is known based on Clause 2.4.5.2(10). The characteristic value for the resistance is lower than its mean value and, for the load, the characteristic value is higher than the mean value.

Individual characteristic values are factored by multiplying or dividing the partial factors. All partial factors specified by EC7 are greater than one and therefore, through factoring, the resistance is diminished and the action amplified. Three sets of partial factors are specified for actions (loadings or loading effects), material properties (e.g. cohesion, friction angle, undrained strength, etc.), and resistances (e.g. bearing resistance, sliding resistance and earth resistance). These three sets of partial factors are not applied simultaneously but there are three design approaches combining different sets of partial factors [7].

- Design approach 1 (DA1): (Combination 1) factoring actions only;
- Design approach 2 (DA2): factoring materials only
- Design approach 3 (DA3): factoring structural actions and materials (geotechnical actions from the soil are unfactored).

According to Eq. (1), the factored resistance (design value of resistance) is required to be greater than or equal to the factored load (design value of load). The design is mainly focused on the ultimate limit state and the serviceability limit state is checked using the design parameters obtained from the ultimate limit state design.

2.2. FORM and intuitive dispersion ellipsoid perspective

The matrix formulation of the Hasofer-Lind reliability index [8] is

\[
\beta = \min_{xF, y} \sqrt{\sum_{i=1}^{n} (x_i - \mu_i) \cdot C^{-1} \cdot (x_i - \mu_i)}
\]

where \( x \) is a vector of the input random variables; \( \mu \) is a vector of the mean values of \( x \); \( C \) is the covariance matrix; \( F \) represents the failure domain. The Hasofer-Lind reliability index was originally proposed for correlated normal random variables. For non-normal distributions, the formulation for \( \beta \) is shown by

\[
\beta = \min_{xF} \sqrt{\sum_{i=1}^{n} \frac{(x_i - \mu_i^N)}{\sigma_i^N}}
\]

where \( R \) is the correlation matrix; \( \mu_i^N \) and \( \sigma_i^N \) are the mean and standard deviation of the equivalent normal distribution of the random variable \( x_i \) and they can be calculated through the transformation by Rackwitz and Flessler [9].

The classical approach to calculate the FORM reliability index involves the rotation of the axes of the original random variables and an iterative scheme to approximate the design point. Low and Tang [10] proposed an intuitive dispersion ellipsoid perspective for FORM analysis as shown in Fig. 1.

The FORM analysis can be described as follows. An ellipse (for the two-dimensional case), tilted for correlated random variables or untitled for uncorrelated random variables, represents the contour line of the joint distribution of two random variables. Finding the design point is equivalent to expanding the ellipse until it touches the limit state surface (LSS) separating the safe domain from the unsafe domain. This tangent point is the design point which is the most probable combination of random variables on LSS. The reliability index \( \beta \) is calculated by \( R/r \), where \( R \) is the distance from the mean value point to the \( \beta \)-ellipse and \( r \) is the distance to the \( 1 \sigma \)-ellipse as shown in Fig. 1. To obtain the reliability index means maximizing the value of the multivariate normal probability density function and is graphically equivalent to finding the smallest ellipse tangent to LSS. This optimization procedure can be implemented in the ubiquitous Excel platform or other software such as MATLAB. An alternative to Eq. (3) is given in Low and Tang [11] as

\[
\beta = \min_{xF} \sqrt{n^T R^{-1} n}
\]

where \( n \) represents the dimensionless vector of \( (x_i - \mu_i^N)/\sigma_i^N \). The constrained optimization method by varying random variables in \( n \)-space (correlated standard normal space) is used to calculate the reliability index \( \beta \) and the probability of failure \( Pf \) can be estimated from

\[
P_f \approx 1 - \Phi(\beta)
\]

where \( \Phi(\cdot) \) is the cumulative distribution function (CDF) of the standard normal distribution. For more details of the Low and Tang method, readers may refer to Low [12,13]. Next, a circular tunnel excavated in the Mohr-Coulomb elastic-perfectly-plastic ground is used to illustrate the difference between the partial factor design and FORM-based design.

2.3. Illustrative example of a circular tunnel

This case study is a circular tunnel in a continuous, homogeneous and isotropic rock mass following the Mohr-Coulomb failure criterion under hydrostatic in situ stress. The analytical solution of the plastic zone size and displacement of the tunnel can be found in Hoek [14]. Four input parameters, namely, friction angle \( \phi \), cohesion \( c \), Young’s modulus \( E \) and the hydrostatic in situ stress \( p_0 \), are treated as random variables while the radius of the tunnel is a deterministic value \( r_0 \approx 2.5 \, \text{m} \), Poisson’s ratio \( \nu = 0.3 \) and the dilation angle \( \alpha \) for the rock mass in this case. The critical support pressure \( p_i^{cr} \), at which the plastic zone starts to develop around the tunnel, is defined by

\[
p_i^{cr} = \frac{2p_0 - c_{om}}{1 + k}
\]

\[
k = \frac{1 + \sin \phi}{1 - \sin \phi}, \quad c_{om} = \frac{2c \cos \phi}{1 - \sin \phi}\frac{1}{1 - \sin \phi}
\]

If the internal support pressure \( p_i \) is greater than the critical value, no plastic zone will appear. For \( p_i \) smaller than \( p_i^{cr} \), the radius of the
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