On the reliability of alternating group graph-based networks

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1. Introduction

The probability of having faults in a multiprocessor computer system increases as the size of the system grows. One way to quantify the reliability of a system is using the probability that a fault-free subsystem of a certain size still exists with the presence of individual faults. The higher the probability is, the more reliable the system is. In this paper, we establish the reliability for networks based on $AG_n$, the $n$-dimensional alternating group graph. More specifically, we calculate the probability of a subnetwork (or subgraph) $AG_{n-1}$ being fault-free, when given a single node's fault probability. Since subnetworks of $AG_n$ intersect in highly complex manners, our scheme is to use the Principle of Inclusion–Exclusion to obtain a lower-bound of the probability, by considering intersections of up to four subgraphs. We show that the lower-bound derived this way is very close to the upper-bound obtained in a previous result, which means the lower-bound we get is a very tight one. Therefore, both lower-bound and upper-bound are close approximations of the accurate probability.
The AG graph is an interconnection network for multicomputer systems. It has been shown to possess many attractive fault tolerant properties, including cycle-embedding [3,17], hamiltonicity [9], vertex pancyclicity [16]. Chiang and Chen [6] showed that the AG isomorphic to arrangement graph An−2, making AGn a special case of the arrangement graph [8].

In this paper, we use the probabilistic fault model to establish the reliability for the subgraphs of AGn. A lower-bound analyzing method for calculating the probability of a subgraph AGn−1 being fault-free will be proposed, where a single-node’s fault probability is given. We will show that the calculated lower-bound is very close to the upper-bound obtained from a previous result [14], which means that the lower-bound we get is a very tight one, and both lower- and upper-bound are very close approximations of the accurate probability.

The rest of this paper is organized as follows. Section 2 introduces some notations used throughout the paper and basic properties of the AG graph. Section 3 summarizes the previous result, and derives the upper-bound of Rn−1(p) and the approximate Rn−1(p). Section 4 presents our main result, which establishes a lower-bound on the reliability for AGn−1, a large subgraph of the AGn. Section 5 points out that the calculated lower-bound is very tight compared with the upper-bound, which is derived from the upper-bound of the arrangement graph obtained in a previous work [14]. Then the comparison result and its implication are discussed. Section 6 summarizes the paper with concluding remarks.

2. Preliminaries

Many large-scale multiprocessor computer systems take interconnection networks as underlying topologies, and an interconnection network is usually represented by a graph G = G(V, E(G)), where the node-set V(G) is the set of processors and the edge-set E(G) is the set of links. For notations and terminologies not defined here, please refer to paper [20]. The alternating group graph was first proposed by Jwo et al. [10] as an interconnection network topology. The definition of the alternating group graph is given as follows.

Let (n) = {1, . . . , n} and x = x1x2 . . . xn where xj ∈ (n) and xj ≠ xi for i ≠ j. Two elements x1 and xj is an inversion of x if xj < xi for i > j. An even permutation is a permutation that contains an even number of inversions. Let Zn be the set of all even permutations over (n). Let s = (12i) and s2 = (12i) for i ∈ {3, . . . , n} on Zn by setting xs−1 (resp., xs) to be the permutation obtained from x by rotating the symbols in positions 1, 2, and i from right to left (resp., left to right).

Definition 1. [10] The n-dimensional alternating group graph, denoted by AGn, is defined as follows:

- The node-set is V(AGn) = Zn.
- The edge-set is E(AGn) = {xy | y = xs−1 or y = xs for some i ∈ {3, . . . , n}.

Fig. 1 depicts the example of AG4, where Z4 = {1234, 1342, 2143, 2314, 2314, 3241, 3241, 3241, 3241, 4132, 4213, 4321} and s−1 = (132) = (1234)1324, s2 = (123) = (1234)2314, s−1 = (142) = (1234)14321, s2 = (124) = (1234)1234. Let

\[ V_{n}^{j,v} = \{ x | x = x_{1}x_{2} . . . x_{j−1}v_{i}x_{j+1} . . . x_{n} ∈ E_{n}\} \]

for j ∈ {3, . . . , n} and v ∈ (n). \[ V_{n}^{j,v} | 1 ≤ v ≤ n \] forms a partition of V(AGn) for a fixed position j ∈ {3, . . . , n}. Let AGn−1,\v represent the subgraph of AGn induced by Vn−1,\v. Then AGn−1,\v is isomorphic to AGn−1. Thus, AGn can be recursively constructed from n copies of subgraph AGn−1. It is easy to check that each AGn−1,\v is a subgraph of AGn, and we say that AGn can be decomposed into n copies of AGn−1 along the jth position.

We can fix m (m > 1) values at m positions to obtain smaller subgraphs of AGn, and denote such an (n − m)-subalternating group graph of AGn as AGn−m. The number of disjoint AGn−m’s in an AGn is \( \binom{n}{m}m! \) for 1 ≤ m ≤ n − 2 and the number of distinct AGn−m is \( \binom{n−2}{m}m! \). Each subalternating group graph can be uniquely labeled as a string of symbols over the set {1, . . . , n, X}, where the symbol X represents all unused digits.

Chiang and Chen [6] showed that the n-alternating group graph AGn is isomorphic to An−2 [8]. The definition of the arrangement graph is given as follows.

Definition 2. [8] Let \( \{ n, . . . , n \} \) and Pn,k be a set of arrangements of k elements in \( \{ n \} \) for n > k. The (n, k)-arrangement graph, denoted by An,k, is defined as follows:

- The node-set is V(An,k) = Pn,k.
- The edge-set is E(An,k) = {xy | x and y differ in exactly one position}.

Let Rn−1(p) be the probability that there exists a fault-free subgraph AGn−1 in an AGn, where p is the probability that a node works. If \( R_{n}^{-1}(p) \) is high (low), then the probability that a operational subgraph of size n − 1 exists is high (low).

We will first present the upper-bound for \( R_{n}^{-1}(p) \) and the approximation of \( R_{n}^{-1}(p) \) by using the known result on the (n, k)-arrangement graph [14]. Then we will establish a lower-bound for \( R_{n}^{-1}(p) \) using the probabilistic fault model and the Principle of Inclusion–Exclusion (PIE) [1].
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