Physica A 392 (2013) 5027–5045

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Analysis of cross-correlations between financial markets after the 2008 crisis ☆,☆☆

A. Sensoy [a,b,*], S. Yuksel [a], M. Erturk [a]

[a] *Borsa Istanbul, Research Department, Resitpasa Mahallesi, Tuncay Artun Caddesi, Emirgan, Istanbul, 34467, Turkey*

[b] *Bilkent University, Department of Mathematics, Ankara, 06800, Turkey*

HIGHLIGHTS

- We use RMT to analyze the cross-correlations between worldwide stock markets.
- We find that the majority of the cross-correlation coefficients arise from randomness.
- We analyze the connection structure of markets before and after the crisis using network theory.
- Key financial markets are revealed.

ARTICLE INFO

Article history:
Received 30 December 2012
Received in revised form 23 April 2013
Available online 25 June 2013

Keywords:
Cross-correlations
Random matrix theory
Complex systems
Minimal spanning tree
Centrality measures

ABSTRACT

We analyze the cross-correlation matrix **C** of the index returns of the main financial markets after the 2008 crisis using methods of random matrix theory. We test the eigenvalues of **C** for universal properties of random matrices and find that the majority of the cross-correlation coefficients arise from randomness. We show that the eigenvector of the largest deviating eigenvalue of **C** represents a global market itself. We reveal that high volatility of financial markets is observed at the same times with high correlations between them which lowers the risk diversification potential even if one constructs a widely internationally diversified portfolio of stocks. We identify and compare the connection and cluster structure of markets before and after the crisis using minimal spanning and ultrametric hierarchical trees. We find that after the crisis, the co-movement degree of the markets increases. We also highlight the key financial markets of pre and post crisis using main centrality measures and analyze the changes. We repeat the study using rank correlation and compare the differences. Further implications are discussed.

1. Introduction

The global financial system is composed of a large variety of markets that are positioned in different geographic locations and in which a broad range of financial products are traded. Despite the diversity of markets, index movements often respond to the same economic announcements or market news [1–3] which implies that financial time series can display similar characteristics and be correlated. Since the work of Markowitz [4], correlations of financial time series are constantly

☆ The views expressed in this work are those of the authors and do not necessarily reflect those of the Borsa Istanbul or its members.

☆☆ An earlier version of this paper was presented at the "Asian Quantitative Finance Conference" organized by the National University of Singapore, January 9–11, 2013 and the "Conference on Mathematics in Finance" organized by the Bank of England and the Institute of Mathematics and its Applications, in Edinburgh, April 8–9, 2013. We thank the conference participants for insightful suggestions.

* Corresponding author at: Borsa Istanbul, Research Department, Resitpasa Mahallesi, Tuncay Artun Caddesi, Emirgan, Istanbul, 34467, Turkey. Tel.: +90 5326959943; fax: +90 2122982189.

E-mail addresses: ahmets@fen.bilkent.edu.tr, ahmet.sensoy@borsaistanbul.com (A. Sensoy).

0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.06.046

a subject of extensive studies both at the theoretical and practical levels. It is important not only for understanding the collective behavior of a complex system but also for asset allocation and estimating the risk of a portfolio.

In particular since the recent 2008 financial crisis, which originated in the US and then spread to almost all markets in the world, many economists have been studying the correlation structure between financial markets and the transmission of volatility from one to another. One of the major difficulties in these studies are the complicated unknown underlying interactions of the financial markets. Besides correlations between markets need not be just pairwise but may rather involve clusters of markets and relationship between any two pair may change in time [5].

In earlier times, physicists experienced similar problems. The problem became popular by Wigner's work in the 1950s for application in nuclear physics, in the study of statistical behavior of neutron resonances and other complex systems of interactions [6]. He tried to understand the energy levels of complex nuclei, when model calculations failed to explain experimental data. To overcome this problem, he assumed that the interactions between the constituents comprising the nucleus are so complex that they can be modeled as random [5]. Based on this assumption, he derived the statistical properties of very large symmetric matrices with i.i.d. entries and the results were in remarkable agreement with experimental data.

More recently random matrix theory (RMT) has been applied to analyze the financial time series [5,7–37]. In particular, correlation matrices are computed for the empirical data and quantities associated with these matrices are compared to those of random matrices. The extent to which properties of the correlation matrices deviate from random matrix predications clarifies the status of the information derived from the computation of covariances [12]. The literature focuses on the correlations between individual stocks in a market; however, in this study we will analyze the cross-correlations between 87 main financial markets in the world by tools of RMT.

The rest of the paper is organized as follows; in Section 2, we give a brief description of the methodology. Section 3 describes the data and contains several results of our analysis; in particular Sections 3.1, 3.2 and 3.4 present the eigenvalue and eigenvector analysis of the correlation matrix with discussion of the relation between volatility and correlation of financial markets. In Section 3.6, we construct a correlation based market network and compare the structure before and after the 2008 financial crisis by tools of graph theory. In Section 4, we use an alternative approach to the construction of the correlation matrix, present the related results and discuss possible further studies. Finally, Section 5 contains some concluding remarks.

2. Methodology

Let $P_i(t)$ be the index of the stock market $i = 1, 2, \ldots, N$ at time t and $t = 0, 1, \ldots, T$. The logarithmic index return of the ith market index over a time interval Δt is given by

$$R_i(t, \Delta t) \equiv \ln P_i(t + \Delta t) - \ln P_i(t). \tag{1}$$

We consider the normalized returns

$$r_i(t) \equiv \frac{R_i - \langle R_i \rangle}{\sigma_i} \tag{2}$$

where $\sigma_i \equiv \sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2}$ is the standard deviation of R_i and $\langle \cdots \rangle$ is the time average over the considered period. Then the equal time cross-correlation matrix C is the matrix with elements

$$c_{ij} \equiv \langle r_i r_j \rangle. \tag{3}$$

In matrix notation, the interaction matrix $\mathbf{C}$ can be written as

$$\mathbf{C} = \frac{1}{T}\mathbf{R}\mathbf{R}^t \tag{4}$$

where $\mathbf{R}$ is an $N \times T$ matrix with entries $r_{im} \equiv r_i(m\Delta t)$ with $i = 1, 2, \ldots, N$; $m = 1, \ldots, T$ and $\mathbf{R}^t$ denotes the transpose of $\mathbf{R}$.

We will compare the properties of the interaction matrix $\mathbf{C}$ with those of a random cross-correlation matrix.

Let $x_i(t)$; $i = 1, 2, \ldots, N$ where $x_i(t)$ are independent, identically distributed random variables. We define the $N \times T$ matrix $\mathbf{A}$ by elements $a_{it} \equiv x_i(t)$. The matrix $\mathbf{W}$ defined as

$$\mathbf{W} = \frac{1}{T}\mathbf{A}\mathbf{A}^t \tag{5}$$

is called a Wishart matrix [38–40]. Let each $x_i(t)$ be normally distributed and rescaled to have zero mean and constant unit standard deviation. Under the restriction of $N \to \infty$, $T \to \infty$ with $Q \equiv T/N > 1$ is fixed, the probability density function $\rho_{rm}(\lambda)$ of eigenvalues λ of the matrix $\mathbf{W}$ is [39,40]

$$\rho_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda} \tag{6}$$