International Review of Financial Analysis xxx (2013) xxx–xxx

Contents lists available at ScienceDirect

International Review of Financial Analysis

Granger-causality in quantiles between financial markets: Using copula approach ☆

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ARTICLE INFO

Available online xxxx

JEL classification:
C5

Keywords:
Contagion in financial markets
Copula functions
Inverting conditional copula
Granger-causality in conditional quantiles

ABSTRACT

This paper considers the Granger-causality in conditional quantile and examines the potential of improving conditional quantile forecasting by accounting for such a causal relationship between financial markets. We consider Granger-causality in distributions by testing whether the copula function of a pair of two financial markets is the independent copula. Among returns on stock markets in the US, Japan and U.K., we find significant Granger-causality in distribution. For a pair of the financial markets where the dependent (conditional) copula is found, we invert the conditional copula to obtain the conditional quantiles. Dependence between returns of two financial markets is modeled using a parametric copula. Different copula functions are compared to test for Granger-causality in distribution and in quantiles. We find significant Granger-causality in the different quantiles of the conditional distributions between foreign stock markets and the US stock market. Granger-causality from foreign stock markets to the US stock market is more significant from UK than from Japan, while causality from the US stock market to UK and Japan stock markets is almost equally significant.

1. Introduction

A causal relationship in a system of economic or financial time series has been widely studied. Following a series of seminal papers by Granger (1969, 1980, 1988), Granger-causality (GC) test becomes a standard tool to detect causal relationship. Granger-causality in mean (GCM) is widely analyzed between macroeconomic variables, such as between money and income, consumption and output, etc. cf. Sims (1972, 1980), Stock and Watson (1989). In financial markets, a growing interest in volatility spill-over promotes the development of Granger-causality tests in volatility. cf. Granger, Robins, and Engle (1986), Lin, Engle, and Ito (1994), Cheung and Ng (1996), Comte and Lieberman (2000). Most tests of Granger-causality assume a bivariate Gaussian distribution and focus on Granger-causality in mean or variance.

A Gaussian distribution cannot capture asymmetric dependence between financial markets. For instance, co-movements between different financial markets behave differently in a bull market and in a bear market. Ang and Chen (2002) assert that non-Gaussian dependence between economic variables or financial variables is prevalent. Associated with the non-elliptical distribution, causality may matter in higher moments or in the dependence structure in a joint density. Thus, it is more informative to test Granger-causality in distribution (GCD) to explore a causal relationship between two financial time series.

We apply a copula-based approach to model the causality and dependence between a pair of two financial time series. Using copula density functions, we construct two tests for GCD. The first test is nonparametric, following Hong and Li (2005), to compare the copula density in quadratic distance with the independent copula density. The second test is parametric; noting that different parametric copula functions imply different dependence structures, we design a method to compare them in an entropy with the independent copula density. Both tests compare out-of-sample predictive ability of copula functions relative to the benchmark independent copula density.

GCD implies Granger-causality in some quantiles. In financial risk management and portfolio management, it is useful to know which quantile leads to the GCD. In particular, Value-at-Risk (VaR) is a quantile in tail that is widely used in capital budgeting and risk control. We are interested in exploring the potential of improving quantile forecasting of a trailing variable Y using information of a preceding variable X. We define Granger-causality in quantile (GCQ), for which quantile forecasts are computed from inverting a conditional copula distribution, and we develop a test for GCQ.

In our empirical application, these copula-based methods are applied to analyze the pair-wise GCD from the Japan stock market to the US stock market (Japan–US), from the UK stock market to the US stock market (UK–US), from the US stock market to the Japan stock market (US–Japan), and from the US stock market to the UK stock market (US–UK).

☆ We thank the editor Catherine Kyrtsou and two anonymous referees, Wolfgang Härdle, Yongmiao Hong, Peter Phillips, and the seminar participants at the Symposium on Econometric Theory and Applications (SETA) for useful comments. We also thank Yongmiao Hong for sharing his code used in Hong and Li (2005), which we modified for copula models in this paper. All errors are our own. A part of the research was started while Lee was visiting the California Institute of Technology. Lee thanks them for their hospitality and the financial support during the visit. Yang is grateful for the Chancellor's Distinguished Fellowship from the University of California, Riverside.
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http://dx.doi.org/10.1016/j.irfa.2013.08.008

We find significant GCD in these four data sets and all sample periods considered (seven different subsample periods), as the benchmark independent copula is clearly rejected in all data sets and subsamples. For GCQ, we compare predictive performance of various copula functions with the benchmark independent copula function over different quantiles of the conditional distribution of one market conditional on another market. It is found that GCQ is significant from US to foreign stock markets and from UK to the US stock market, but not from Japan to US. The result is robust over the seven subsamples.

The rest of the paper is organized as follows. Section 2 introduces two tests of GCD based on copula density functions. Both tests are based on the distance measures and thus measure the strength of GCD. Section 3 defines GCQ and develops a method to test for GCQ. Section 4 reports empirical findings on GCD and GCQ. Section 5 concludes. Appendix A reviews some basic results on copula functions.

2. Granger-causality in distribution

In this section, we define GCD and introduce two statistics (based on the information entropy) which measure the strength of the GCD. The data used in our empirical applications are the daily return on the S&P500 stock index (S&P500), the NIKKEI 225 stock index (NIKKEI) and FTSE 100 stock index (FTSE). On the same trading day t, trading in the Tokyo Stock Exchange and London Stock Exchange precedes that in the New York Stock Exchange. To explore the causality between two financial markets, we use $\{X_t\}$ to denote the preceding variable and $\{Y_t\}$ as the trailing variable. For instance, $\{X_t\}$ denotes stock returns on the NIKKEI and $\{Y_t\}$ denotes stock returns on the S&P500. See Table 1 (Panel A). We are only interested in causality in the same day or in the next day. Causality may occur in a longer time horizon. However, Dufour and Renault (1998) and Dufour, Pelletier, and Renault (2006) show that in the financial market, if there is non-causality between X_t and Y_t, it will be difficult to explore Granger-causality in a longer horizon. With the development of information technology, impact of information in one market has the most significant effects in a short period, and we focus on causality in daily frequency.

Using a copula-based approach, various dependence structures can be flexibly modeled by a copula and marginal distribution functions. Dependence measures, such as Kendall's τ and Spearman's ρ, can also be easily computed using a copula function. Therefore, recently copula models have been widely used to model dependence between financial time series. Some recent research includes Li (2000), Scaillet and Fermanian (2003), Embrechts, Hoing, and Juri (2003), Patton (2006a, b), Granger, Teräsvitra, and Patton (2006), and Chen and Fan (2006a, 2006b), among others. We refer to Appendix A for more details. In this paper, we show how to use copula functions to test for GCD, how to measure the degree of GCD from using the log-likelihood of the copula density functions, and how to invert the conditional copula distribution functions to forecast the conditional quantiles which enable us to test for GCQ.

We use the following notation. Let R denote the sample size for estimation (for which we use a rolling scheme), P the size of the out-of-sample period for forecast evaluation, and $T = R + P$. Suppose the stock market X closes before the stock market Y closes. Let $\mathcal{G}_t$ be the information set before the stock market X closes and let $\mathcal{F}_t$ be the information set after the stock market X closes but before the stock market Y closes, i.e., $\mathcal{F}_t = \mathcal{G}_t \cup \{x_t\}$. Consider the conditional distribution functions, $F_X(x|\mathcal{G}_t) = \Pr(X_t < x|\mathcal{G}_t)$, $F_Y(y|\mathcal{G}_t) = \Pr(Y_t < y|\mathcal{G}_t)$, and $F_{XY}(x,y|\mathcal{G}_t) = \Pr(X_t < x \text{ and } Y_t < y|\mathcal{G}_t)$. Let $f_X(x|\mathcal{G}_t)$, $f_Y(y|\mathcal{G}_t)$, and $f_{XY}(x,y|\mathcal{G}_t)$ be the corresponding densities. Let $U_t = F_X(X_t|\mathcal{G}_t)$ and $V_t = F_Y(Y_t|\mathcal{G}_t)$ be the (conditional) probability integral transforms (PIT) of X_t and Y_t. Let $C(u,v)$ and $c(u,v)$ be the conditional copula function and the conditional copula density function respectively. See Appendix A for a brief introduction to the copula theory. We define GCD as follows.

Definition 1. (Non-Granger-causality in distribution, NGCD): $\{X_t\}$ does not Granger-cause $\{Y_t\}$ in distribution if and only if $\Pr(Y_t < y|\mathcal{F}_t) = \Pr(Y_t < y|\mathcal{G}_t)$ a.s. for all y.

There is GCD if $\Pr(Y_t < y|\mathcal{F}_t) \neq \Pr(Y_t < y|\mathcal{G}_t)$ for some y. $\{X_t\}$ does not Granger-cause $\{Y_t\}$ in distribution if $F_Y(y|\mathcal{F}_t) = F_Y(y|\mathcal{G}_t)$ *a.s.* This implies that testing for NGCD can be based on the null hypothesis

$$H_0^1 : f_Y(y|\mathcal{F}_t) = f_Y(y|\mathcal{G}_t). \tag{1}$$

Note that the joint density is a product of the conditional density and the marginal density

$$f_{XY}(x,y|\mathcal{G}_t) = f_Y(y|\mathcal{F}_t) \times f_X(x|\mathcal{G}_t) \tag{2}$$

and a joint density can be written from the decomposition theorem in Eq. (43) as

$$f_{XY}(x,y|\mathcal{G}_t) = f_X(x|\mathcal{G}_t) \times f_Y(y|\mathcal{G}_t) \times c(u,v). \tag{3}$$

From Eqs. (2) and (3), we obtain

$$f_Y(y|\mathcal{F}_t) = f_Y(y|\mathcal{G}_t) \times c(u,v). \tag{4}$$

Hence, the null hypothesis of NGCD, H_0^1 in Eq. (1), can be stated as the null hypothesis that the copula density is the independent copula,

$$H_0^2 : c(u,v) = 1. \tag{5}$$

Table 1
Description of data sets and subsamples.

Panel A. Data sets

	X	Y	Observations
Data Set 1A (Japan–US)	Returns on NIKKEI 225	Returns on S&P500	2566
Data Set 1B (US–Japan)	Returns on S&P500	Returns on NIKKEI 225 for next day	2566
Data Set 2A (UK–US)	Returns on FTSE 100	Returns on S&P501	2566
Data Set 2B (US–UK)	Returns on S&P500	Returns on FTSE 100 for next day	2566

Panel B. Subsamples in each date set

	Starting date	Ending date	T	R	P
Subsample 1	Jan-95	Dec-99	1172	706	466
Subsample 2	Jan-96	Dec-00	1171	702	469
Subsample 3	Jan-97	Dec-01	1164	699	465
Subsample 4	Jan-98	Dec-02	1163	703	460
Subsample 5	Jan-99	Dec-03	1162	697	465
Subsample 6	Jan-00	Dec-04	1162	697	465
Subsample 7	Jan-01	Dec-05	1157	693	464

Please cite this article as: Lee, T.-H., & Yang, W., Granger-causality in quantiles between financial markets: Using copula approach, *International Review of Financial Analysis* (2013), http://dx.doi.org/10.1016/j.irfa.2013.08.008