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A DC programming heuristic applied to the logistics network design problem

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ABSTRACT

This paper proposes a new heuristic method for the logistics network design and planning problem based on linear relaxation and DC (difference of convex functions) programming. We consider a multi-period, multi-echelon, multi-commodity and multi-product problem defined as a large scale mixed integer linear programming (MILP) model. The method is experimented on data sets of various size. The numerical results validate the efficiency of the heuristic for instances with up to several dozens facilities, 18 products and 270 retailers.

1. Introduction

An efficient configuration of a logistics network must enable the production and delivery of goods to customers at the lowest cost while satisfying a required service level. This topic has been the subject of numerous optimisation models and methods in the fields of operations management and operations research. Optimisation of supply chain design and strategic planning problems are often modelled by mixed integer linear programmes (MILPs). In these models, the location issues are often represented by binary variable decisions while the product flows along the logistics network are represented by continuous decision variables.

Several reviews have been published in the last 10 years. Beamon (1998) distinguishes models with deterministic data from those with stochastic data. Owen and Daskin (1998) clearly separate the static and dynamic models and list various supply chain performance measures. Sahin and Süral (2007) present a wide range of applications. Finally, Daskin et al. (2005) and Melo et al. (2009) propose an extensive review of location problems in the context of supply chain design and planning.

The present paper proposes an enhanced version of an LP-rounding heuristic for the design and planning of complex supply chains. The idea of LP-rounding is to solve the linear relaxation of some MILPs and to round the fractional variables in order to recover integer feasible solutions. Despite its poor formal guarantee of performance, linear relaxation is known to yield good lower bounds for some assignment or location problems (Benders and van Nunen, 1983). LP-rounding methods have been recently applied to the general assignment problem (French and Wilson, 2007) or lot-sizing problems (Hardin et al., 2007). In the field of facility location or logistics network planning, linear relaxation-based heuristics have been used for the single facility location and more complex models (Levi et al., 2004).

In a recent paper, Thanh et al. (2010) proposed an LP-rounding method combined with correction procedures for the strategic planning of complex logistics networks. The method relies on successive linear relaxations of the original MILP. At each iteration, some facility location variables are set at 0 or 1, either directly by the linear relaxation or by some rounding procedures. The method produces a sequence of MILPs of decreasing size. After a large enough number of iterations, the resulting MILP can be eventually handled by a solver. The heuristic is able to produce feasible solutions for every tested instance thanks to efficient correction procedures. The observed average reduction in computation time compared to the solver is around 80% while the loss in the objective function is 1.5%. However, for larger instances, the size of the residual MILP at the last step hinders the efficiency of the heuristic.

The proposed improvement consists of integrating a DC programming step into this method. At each iteration, the optimisation problem is modelled as a DC (difference of convex functions) problem and solved with the DC algorithm (DCA). This algorithm is used as a local search to improve the current upper bound. The numerical experiments show that introducing this DC formulation to solve the relaxed problem speeds up the heuristic. It also yields solutions that rarely require the use of the correction procedures.

The remainder of this paper is organised as follows. Section 2 summarises the underlying mathematical model, which is extensively detailed in Appendix A. Section 3 recalls the main principles of DC programming and the DC algorithm. In Section 4, we introduce

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the DC-based heuristic and show how to apply it to the logistics network design problem. Numerical results are presented in Section 5.

2. A brief description of the model

We consider a supply chain composed of four layers: potential first tier suppliers, manufacturing centres (plants), distribution centres and retailers or final customers. These layers are represented in Fig. 1.

The logistics network is dedicated to the production and distribution of several products manufactured in one or several steps. The model under consideration can be viewed as a facility location problem with multiple layers, multiple commodities, multiple periods and additional constraints. The complete description of the MILP has been proposed in Thanh et al. (2008) and can be found in Appendix A. In this section, we only outline its main components. The objective function to be minimised is the sum of all the logistics costs over the time horizon considered. This includes five fixed costs (supplier selection, cost of opening, closing or enlarging facilities, cost of operating facilities) and four variable costs (production or subcontracting the production, storage, transportation between facilities). All these costs are linear and, without loss of generality, they are considered as time-independent.

The decision variables include binary variables that model strategic decisions and continuous variables that model product flow along the network.

The binary decision variables are the following:

- $x_i^t = 1$ if facility i is active at period t. Depending on the context, one variable x_i^t can represent a supplier, a plant or a distribution centre. A supplier is said to be active if it delivers at least one raw material. A plant or a distribution centre is active if it is open in the corresponding period. As shown in Thanh et al. (2008), variables x_i^t, associated with plants and distribution centres, are the core of the problem and have a direct influence on a large part of the cost function.
- $y_{i,o}^t = 1$ if facility i is enlarged at period t by adding some optional capacity o. The optional capacities are modular extensions that may be appended to existing production or distribution centres in order to increase their capacity. For each facility, there is a list of available sizes of optional capacities.
- Variables $z_{s,p}^t$ and v_s^t concern supplier selection and supplier discounted costs. Since the heuristic focuses on location variables, we do not detail them.

The continuous decision variables are the following:

- $q_1^t(p,i,j)$ is the quantity of product p sent from facility i to facility j at period t,
- $q_2^t(p,i)$ is the quantity of product p produced in plant i at period t,
- $q_3^t(p, i)$ is the quantity of product p subcontracted by plant i at period t,
- $q_4^t(p,j)$ is the quantity of product p held in distribution centre j at period t.

The set of constraints models realistic rules for the management of the whole network. It can be divided into four categories: demand satisfaction constraints, capacity limitation constraints, coherence constraints (for example only open facilities can produce) and integrity and non-negativity constraints.

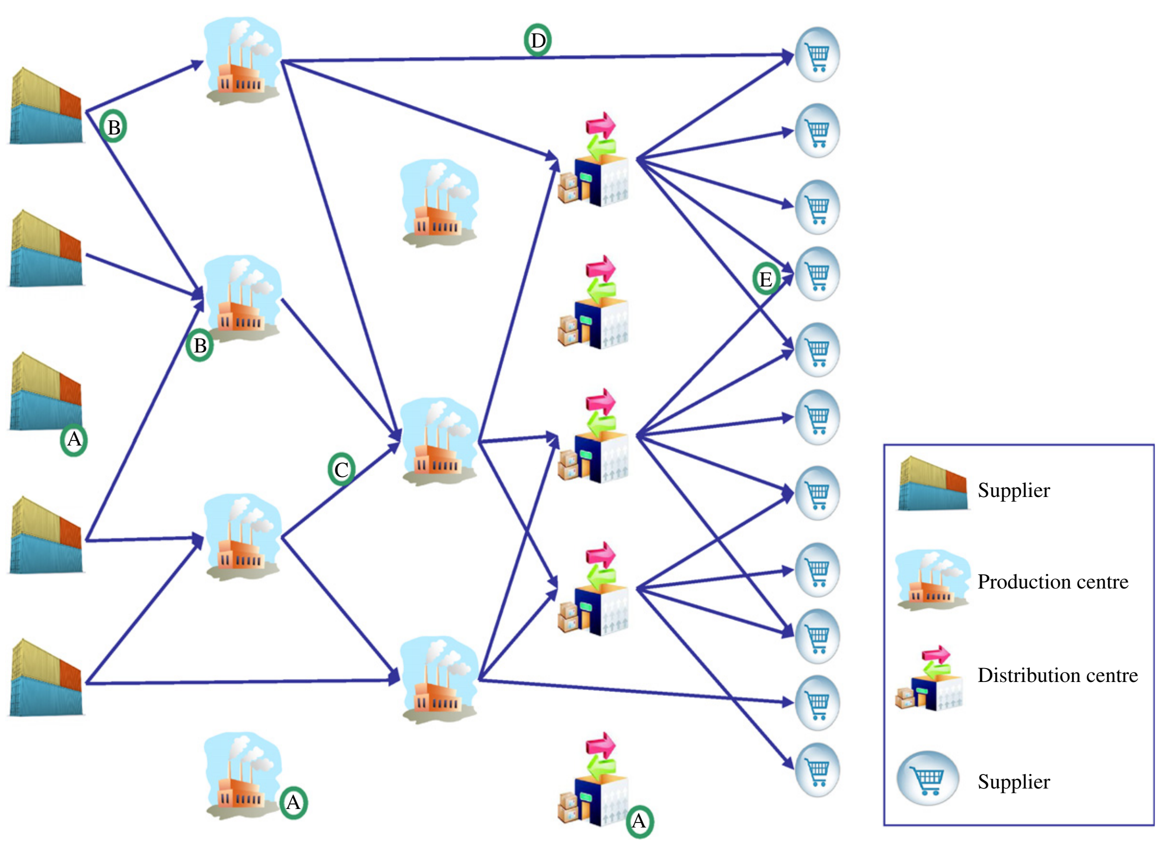

Fig. 1. The logistic network considered. (A) Some facilities may not be used. (B) Suppliers can deliver several manufacturing centres and manufacturing centres can be delivered by several suppliers. (C) Production of goods can be split among various manufacturing centres. In this case there must exist a link between the corresponding facilities. (D) There can exist direct deliveries from manufacturing centres to some important customers. (E) The customers can be delivered by various distribution centres (no single sourcing constraint).