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Multifractal diffusion entropy analysis on stock volatility in financial markets

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ABSTRACT

This paper introduces a generalized diffusion entropy analysis method to analyze long-range correlation then applies this method to stock volatility series. The method uses the techniques of the diffusion process and Rényi entropy to focus on the scaling behaviors of regular volatility and extreme volatility respectively in developed and emerging markets. It successfully distinguishes their differences where regular volatility exhibits long-range persistence while extreme volatility reveals anti-persistence.

1. Introduction

In recent years, there has been increasing concern among researchers, practitioners and regulators over how to evaluate financial risk. Volatility, usually measured by the standard deviation of portfolio returns, is uniquely important in financial markets, which is often taken to represent the portfolio's risk. Consequently, the literature on volatility study is sizeable and still growing. There has been rapid development of techniques for measuring and managing financial risk, partially motivated by a spate of recent financial disasters involving derivative securities [1–8]. Financial market volatility is indispensable for asset and derivative pricing, asset allocation, and risk management. A model-free measure of volatility is the sample variance of returns. It is commonly used to sample the intraday data on a daily basis so that closing prices are recorded, from which daily returns are subsequently computed. Using daily data, the returns may be freely estimated spanning over any number of days and, as such, one can construct a time series of model-free variance estimates. When one chooses the observation frequency of this series, an important trade-off has to be made, however. When the variances are calculated using a large number of observations, many interesting properties of volatility tend to disappear (the volatility clustering and leverage effect, for instance). On the other hand, if only very few observations are used, the measures are subject to great error. For these, the definition of volatility is often termed through the absolute value of logarithmic return as follows:

$$r(t) = \log S(t) - \log S(t-1) \quad \text{and} \quad R(t) = |r(t)| \tag{1}$$

where $S(t)$, $r(t)$, $R(t)$ respectively refer to the daily closing price at time t, the return and the volatility. In many cases for better comparisons among different areas, nations or markets, $R(t)$ is transformed through standardization, i.e. $R'(t) = [R(t) - \langle R(t)\rangle]/\sigma(R(t))$, where $\langle\cdots\rangle$ and σ represent the mean value and standard deviation respectively.

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Recently, the long-range correlation detection of multifractal time series has attracted more attention and has become an active area of research for statistical physicists. Many traditional techniques and newly developed methods were used to explore possible scaling behavior of stock return and volatility in developed as well as emerging markets [9–18]. Relative researches focused on long-range correlation which provided possibilities to gain insight into the complex, irregular, chaotic and vulnerable (to policies) financial series characterization and their long-range pattern of evolution, and offered approaches to minimize risk as well as predict their future dynamical trend. A frequently used approach to quantitatively estimate the correlation of stationary time series $\{x(i)\}$, $i = 1, 2, \ldots, N$ is the correlation function $C(s) = \sum_{i=1}^{N-s}[x(i) - \bar{x}][x(i+s) - \bar{x}]/{\sigma_x}^2$. N refers to the series length, s refers to the time delay, $\bar{x}$ denotes the global average and ${\sigma_x}^2$ denotes the standard deviation. The series is termed as long-range correlated if $C(s) \sim s^{-\gamma}$ and correlation exponent γ satisfies $0 < \gamma < 1$, in which case the mean correlation time $T = \sum_{s=1}^{\infty} C(s)\mathrm{d}s$ diverges for infinitely long series. The correlation exponent or so-called scaling exponent, as expected, plays a crucial role in estimating the long-range correlation. However, for nonstationary time series the average $\bar{x}$ is not well-defined thus the definition of $C(s)$ gets problematic. Other alternative methods for stationary fractal time series include spectral analysis, Hurst's rescaled-range analysis, fluctuation analysis, etc.

However, real-world complex dynamics often exhibit anomalous, chaotic, irregular and nonstationary characterizations integrated by varieties of factors including internal drive and external disturbance. The selection of an appropriate technique is crucial for accurate analysis especially for a real financial system. The methods currently used to determine the scaling exponent of a complex dynamic process described by a nonstationary time series mainly include: (i) the wavelet analysis deriving from signal theory and frequency decompositions [19,20]; (ii) the detrended fluctuation analysis constructed on random walk and detrended variance [21,22]; (iii) the diffusion entropy analysis (DEA) based on diffusion process and Shannon entropy [23–25]. The advantage of the DEA method is that it can determine the correct scaling exponent even when the statistical properties are anomalous or strange kinetics are involved. In this paper, the DEA method is extended to the multifractal case then applied to distinguish the differences of long-range scaling behaviors between regular volatility and extreme volatility. The underlying ideas behind such application include: (i) the volatility series shows a clustering phenomenon, which can be considered as the basic element of regular pattern; (ii) extreme volatility really plays a crucial role in risk management and selection of portfolio.

The paper is organized as follows. In Section 2, we introduce the concept of Rényi entropy and propose the multifractal diffusion entropy analysis (MDEA). In Section 3, we study the long-range scaling behaviors of standard volatilities of the developed market and emerging market for empirical analysis. A brief conclusion is presented in Section 4.

2. Multifractal diffusion entropy analysis

2.1. Rényi entropy

Entropy plays an important role in statistical mechanics [26–28] and information theory [29,30]. In the last decade much attention has been devoted to this subject. Since the introduction of the thermodynamic concept of entropy and its description in probabilistic terms by Boltzmann, the successful probabilistic description of natural systems has transformed entropy as a fundamental branch of modern science. Abundant references introduce different types of entropy [31–33], such as Shannon, Rényi and Tsallis entropy. The additive Shannon entropy plays a central role in extensive statistical mechanics, whereas the nonadditive Tsallis entropy is the core for nonextensive statistical mechanics.

In 1948, Shannon used some axioms to introduce Shannon entropy [34]. Subsequently, a number of sets of axioms were proposed by Khinchin and others [35,36]. Shannon entropy H can still be used as a magnitude in a general situation with N accessible states:

$$H = -\sum_{i=1}^{N} p_i \log p_i \tag{2}$$

with p_i the associated probability, $\sum_{i=1}^{N} p_i = 1$. Extension of Shannon's original work has resulted in many alternative measures of information entropy. By relaxing the additivity requirement, Rényi generalized Shannon entropy to a one-parameter family of entropies by defining an entropy of order q which is called the Rényi entropy [37]. The Rényi entropy is used for measuring biological diversity and studying fractals and multifractals [38] in the form

$$H_q = \frac{1}{1-q} \log \sum_{i=1}^{N} p_i^q \tag{3}$$

where q is an index or weight of different probabilities. Large q makes H_q focus on the extreme events with small probability while small q makes H_q focus on the regular events with large probability. Shannon information H is obtained in the limit q infinitely close to 1, $H = \lim_{q\to 1} H_q = -\sum_{i=1}^{N} p_i \log p_i$. The concept of Rényi entropy has a number of applications in coding theory, statistical mechanics and related fields, and other areas, which is frequently applied for discussing fractal and multifractal systems [39,40].