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On the scaling of the distribution of daily price fluctuations in the Mexican financial market index

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ABSTRACT

In this paper, a statistical analysis of log-return fluctuations of the IPC, the Mexican Stock Market Index is presented. A sample of daily data covering the period from 04/09/2000–04/09/2010 was analyzed, and fitted to different distributions. Tests of the goodness of fit were performed in order to quantitatively asses the quality of the estimation. Special attention was paid to the impact of the size of the sample on the estimated decay of the distributions tail. In this study a forceful rejection of normality was obtained. On the other hand, the null hypothesis that the log-fluctuations are fitted to a α-stable Lévy distribution cannot be rejected at the 5% significance level.

1. Introduction

Today it is widely acknowledged that for the proper management of assets and prices (and the related investment risks) proper modeling of the return distribution of financial assets is required. For instance, the answer to whether it is possible to beat the market except by chance depends on whether stock market prices display long memory and how probable are very large price fluctuations. The crucial difficulty is, however, that the financial market is a very complex system; it has a large number of non-linearly interacting internal elements, and is highly sensible to the action of external forces. Even more, the real challenge here is that the number of the system constituents, and the details of their interactions and of the external factors acting upon it are actually barely known.

Physicists have a long tradition of dealing with similar systems. The statistical description of systems of many particles was developed in parallel with the statistical analysis of market dynamics [1–3]. For instance, taking into account the wide applicability of the Central Limit Theorem, Bachelier assumed that the return over a given time scale is the consequence of many independent "microscopic" events, which then lead to a normal distribution of returns. Thus, he modeled their dynamics as an uncorrelated random walk with independent, identically Gaussian distributed random variables, i.e., as a Brownian motion [2]. Since then, the Gaussian assumption for the distribution of returns has been frequently used in mathematical finance and is one of the key assumptions behind the classic Black–Scholes option pricing formula [4], which is based on a Wiener process in the continuous-time setting or on appropriate discrete-time versions such as binomial trees.

Thought the simplifications the normal distribution provides in analytical calculation are very valuable, empirical studies show that the distribution of returns has a tail heavier than that of a Gaussian [5–13]. To illustrate this fact, we show in Fig. 1 the histogram for daily logarithm differences of the Mexican IPC index from April 9th, 2000 to April 9th, 2010. Clearly, large events are very frequent in the data, a fact largely underestimated by a Gaussian process and of utmost importance

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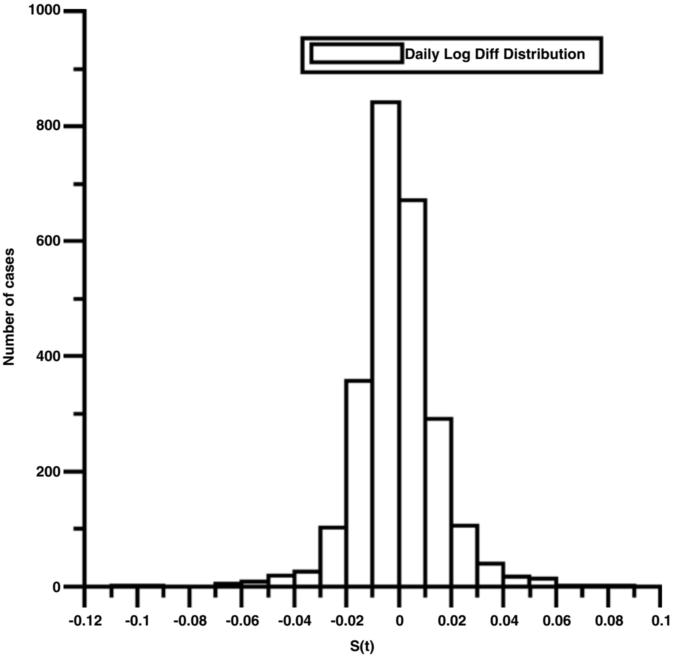

Fig. 1. Histogram for daily logarithm differences of the Mexican IPC index from April 9th, 2000 to April 9th, 2010.

for financial management. It is remarkable that such a feature is present in quite different markets, signaling a possible universal behavior. Such a signal is also supported by the study of the statistical properties of other measures of how much the market is likely to fluctuate. For instance, in Refs. [14–16] similar power-law behavior were respectively reported for the correlations, autocorrelations and cross-correlations of the US market volatility.

One of the skills of physicists is the search for universal laws, i.e., common features in most particular realizations of a general class of phenomena. This can be done even if the "microscopic" details distinguishing each case are not fully understood. Heavy tailed distributions are commonly described by a power law (at least in a range of scales), which in turn implies scale invariance, a distinct signature of fractals. Fractals have been shown to be a common geometrical pattern in many natural systems. Finally, many of these systems may be in a state of self-organized criticality, a paradigm that would explain how organization arises in complex systems and makes them more predictable. This could be also the case for other dynamical systems outside the realm of natural sciences.

In his pioneering analysis of cotton prices, Mandelbrot [5] (the founder of fractal geometry) observed that in addition to being non-Gaussian, the process of returns shows another interesting property: time scaling, that is, the distributions of returns for various choices of t, ranging from one day up to one month have similar functional forms. As it was already mentioned, observed stock market prices are assumed to be the sum of many small terms, hence a statistical model to describe them must be such that the sum of two independent random variables having the given distribution (with a parameter α describing the decay of the tail) yields again the same kind of distribution (with the same value of α). Motivated by these empirical findings and reasoning, Mandelbrot proposed that the returns can be modeled as a kind of stable process introduced by Lévy in 1925 [3] (for details about this distribution and Mandelbrot's connection with market behavior see also Ref. [17]). Lévy stable distributions are attractive because they are supported by the generalized Central Limit theorem. The theorem states that stable laws are the only possible limit distributions for properly normalized and centered sums of independent, identically distributed random variables.

An issue here is whether the underlying distributions are actually stable. Stability only holds for $\alpha \in (0, 2]$ and some authors have found that the tails of some financial time series have to be modeled with $\alpha > 2$. For instance, in Ref. [11] the distributions of return fluctuations of the *S&P* 500, NIKKEI and Hang-Seng indices were found to be consistent with a power-law asymptotic behavior, characterized by an exponent $\alpha \approx 3$, well outside the stable Lévy regime. Similar results were reported for the German DAX index [10] and the Mexican IPC [13]. Conclusive results on the distribution of returns are difficult to obtain, and require a large amount of data to study the rare events that give rise to the fat tails [18].

Another issue is that according to Cont [12], in order for a parametric distributional model to reproduce the properties of the empirical distribution it must have at least four parameters: a location parameter, a scale parameter, a parameter describing the decay of the tails and an asymmetry parameter. And we know of many other heavy-tailed alternative distributions (such as student's t, hyperbolic or normal inverse Gaussian) which fulfill this condition. Moreover, there are several variants of the Lévy distribution and some of them have been also successfully used for fitting financial data, like the asymmetric Lévy flights [19] and the truncated distribution [20].