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Market value of life insurance contracts under stochastic interest rates and default risk☆

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Abstract

The purpose of this article is to value some life insurance contracts in a stochastic interest rate environment taking into account the default risk of the underlying insurance company. The participating life insurance contracts considered here can be expressed as portfolios of barrier options as shown by Grosen and Jørgensen [J. Risk Insurance 64 (3) (1997) 481–503]. In order to price these options, the Longstaff and Schwartz [J. Finance 50 (3) (1995) 789–820] methodology is used with the Collin-Dufresne and Goldstein [J. Finance 56 (5) (2001) 1929–1957] correction.

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0. Introduction

Life insurance companies offer complex contracts written with the following many covenants: interest rate guarantees, bonus and surrender options, equity-linked policies, choice of a reference portfolio, participating policies. Each particular covenant has a value and is part of the company liabilities. These embedded options should not be ignored and must be priced. Many life-insurance companies, having neglected them for a long time, increased the difficulties they faced in the 1990s.

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Most of the recent studies rely on the Briys and de Varenne (1997a, 1997b) model. These authors aim at providing a fair valuation of liabilities. By this, it is meant that market value is the reference. More precisely, the computed prices must be arbitrage free. The life insurance contracts are thus considered as purely financial assets traded on a liquid market among perfectly informed investors. This fact is taken as a fundamental assumption in these studies, and it is the basic hypothesis we make in this article. Note that this principle is in line with the Financial Accounting Standards Board (FASB) and International Accounting Standard Board (IASB) directives.

Although Briys and de Varenne (1994, 1997a, 1997b) work in continuous time, their model is essentially a single-period one, and furthermore does not take into account the mortality risk. They value the assets and liabilities of an insurance company which sells only one type of contract. The default can occur only at maturity. Their framework is of the Merton type, and they can therefore obtain closed-form formulae which permit to adjust the different parameters involved in a fair contract. Nevertheless, this model can be considered as a prototype in the valuation of life insurance contract.

Miltersen and Persson (2003) propose a multi-period extension and also provide closed form formulae. Bacinello (2001) analyzes the most sold life insurance contract in Italy. She takes into account mortality and suggests a contract which offers the choice among different triplets of technical rate, participation level and volatility. Paying each year a premium, the insured customer gets the guarantee to recover his initial investment accrued at a fixed rate and can possibly benefit from a bonus indexed on a reference portfolio. The pricing is achieved under the standard Black and Scholes model and assuming independence between mortality risk and financial risk.

Tanskanen and Lukkarinen (2003) consider general participating life insurance contracts. Their contract values depend on the evolution of a reference portfolio at different dates. These authors incorporate the following features: minimum interest rate guaranteed each year, right to change each year the reference portfolio, as well as possibility to surrender each year the contract—giving it a Bermudian aspect. They work with constant interest rates and a constant volatility.

Because there are various kinds of contracts and modeling frameworks, the pricing methodologies are diverse. In fact, mortality, a stochastic interest rate environment and stochastic volatilities, for instance, can be taken into account as well as the right to sell back the contract. Participating policies are also multiple. It must be noted that closed form solutions are obtained in the simple Black and Scholes setting. Tanskanen and Lukkarinen (2003) use a numerical procedure to solve their partial differential equation in order to compute the surrender option.

Jørgensen (2001) and Grosen and Jørgensen (2002) show that a life insurance contract with a minimum interest rate guarantee can be expressed in four terms, the final guarantee (equivalent to a zero-coupon bond), the European bonus option associated with a percentage of the positive performance of the company's asset portfolio, if any, a put option linked to the default risk, and finally a fourth term which is a rebate given to the policyholders in case of default prior to the maturity date.

In Grosen and Jørgensen (1997), the possibility of an early payment is envisaged. To treat this American-style contract they use a binomial lattice whereas Jensen et al. (2001) use a finite difference approach. Grosen and Jørgensen (2002) take into account a default barrier of an exponential type. They obtain closed form formulae in the case of constant interest rates. Jørgensen (2001) extends this study to the more difficult case of stochastic interest rates, using a Monte-Carlo approach.

This study is devoted to the valuation of life insurance contracts in the presence of a stochastic term structure of interest rates, it also takes into account the company's default risk. We provide an alternative method to trees, numerical solutions of PDE and Monte-Carlo simulations, schemes usually used to price such contracts. The term structure of interest rates considered here stems from the classical Heath et al. (1992) framework. Amongst the two standard choices of zero-coupon volatilities making the instantaneous risk-free rate Markovian – linear volatility as in the Ho and Lee model or exponential volatility as in the Hull and White model – we take the second one. Our model is therefore a Vasicek one. Note that we could have considered in our paper a full Hull and White or generalized Vasicek framework by relying on a purely exogenously specified (by a set of zero-coupons) initial term structure of interest rates. The extension of our computations to a Ho and Lee choice of zero-coupon volatility is also straightforward. Our valuation method relies on Collin-Dufresne and Goldstein's (2001) article which is an