Original article

Generalized semi-open and pre-semiopen sets via ideals

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Abstract

In this paper we have introduced a new type of sets termed as \( \mu^* \)-open sets which unifies semiopen sets, \( \beta \)-open sets and discussed some of its properties. We have also introduced another type of weak open sets termed as \( I_\mu \)-open sets depending on a GT as well as an ideal on a topological space. Finally the concept of weakly \( I_\mu \)-open sets are investigated.

Keywords: \( \mu \)-open set; Ideal; \( \mu^* \)-open set; \( I_\mu \)-open set; Weakly \( I_\mu \)-open set

1. Introduction

The concept of ideal on topological spaces was studied by Kuratowski [1] and Vaidyanathaswamy [2] which is one of the important areas of research in the branch of mathematics. After them different mathematicians applied the concept of ideals in topological spaces (see [2–8]). In the past few years mathematicians turned their attention towards the generalized open sets (see [8–12] for details). Our aim in this paper is to use the concept of ideals in the generalized topology introduced by A. Császár. We recall some notions defined in [10].

Let \( \text{exp} X \) denote the power set of a non-empty set \( X \). A class \( \mu \subseteq \text{exp} X \) is called a generalized topology [10], (briefly, GT) if \( \emptyset \in \mu \) and \( \mu \) is closed under arbitrary union. The elements of \( \mu \) are called \( \mu \)-open sets and the complement of \( \mu \)-open sets are known as \( \mu \)-closed sets. A set \( X \) with a GT \( \mu \) on it is known as a generalized topological space (briefly, GTS) and is denoted by \((X, \mu)\). A GT \( \mu \) is said to be a quasi topology (briefly QT) [17] if \( M, M' \in \mu \) implies \( M \cap M' \in \mu \). The pair \((X, \mu)\) is said to be a QTS if \( \mu \) is a QT on \( X \).

For any \( A \subseteq X \), the generalized \( \mu \)-closure of \( A \) is denoted by \( c_\mu (A) \) and is defined by \( c_\mu (A) = \cap \{ F : F \in \mu \text{ and } A \subseteq F \} \), similarly \( i_\mu (A) = \cup \{ U : U \subseteq A \text{ and } U \in \mu \} \) (see [10,11]). Throughout the paper \( \mu, \lambda \) will always mean GT on the respective sets.

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An ideal [1] \( I \) on a topological space \((X, \tau)\) is a non-empty collection of subsets of \(X\) with the following properties:

(i) \( A \subseteq B \) and \( B \in I \Rightarrow A \in I \)

(ii) \( A \in I, B \in I \Rightarrow A \cup B \in I \).

An ideal \( I \) on a topological space \((X, \tau)\) is denoted by \((X, \tau, I)\) and known as an ideal topological space.

2. \( \mu^* \)-open sets

**Definition 2.1.** Let \( \mu \) be a GT on a topological space \((X, \tau)\). A subset \( A \) of \( X \) is called \( \mu^* \)-open [13] if \( A \subseteq cl(i_\mu(A)) \).

**Theorem 2.2.** Let \( \mu \) be a GT on a topological space \((X, \tau)\). Then \( A \) is \( \mu^* \)-open if and only if there exists a \( \mu \)-open set \( U \) such that \( U \subseteq A \subseteq cl(U) \).

**Proof.** Let \( A \) be a \( \mu^* \)-open set. Then \( A \subseteq cl(i_\mu(A)) \). Let \( U = i_\mu(A) \). Then \( U \) is \( \mu \)-open and \( U \subseteq A \subseteq cl(i_\mu(A)) = cl(U) \). Conversely, let there exist a \( \mu \)-open set \( U \) such that \( U \subseteq A \subseteq cl(U) \). Then \( U \subseteq A \Rightarrow U \subseteq i_\mu(A) \Rightarrow cl(U) \subseteq cl(i_\mu(A)) \Rightarrow A \subseteq cl(i_\mu(A)) \). Thus \( A \) is \( \mu^* \)-open.

**Remark 2.3.** Let \( \mu \) be a GT on a topological space \((X, \tau)\). If

(i) \( \mu = \tau \), then \( \mu^* \)-open set reduces to semiopen set [14];

(ii) \( \mu = P O(X) \), then \( \mu^* \)-open set reduces to \( \beta \)-open set [15];

(iii) every \( \mu \)-open set is \( \mu^* \)-open;

(iv) If \( \lambda \) be any other GT on \( X \) with \( \mu \subseteq \lambda \), then every \( \mu^* \)-open set is \( \lambda^* \)-open.

**Note 2.4.** Let \( \mu \) be a GT on a topological space \((X, \tau)\). Then the collection of all \( \mu^* \)-open sets forms a GT on \( X \).

**Proof.** Clearly \( \emptyset \) is a \( \mu^* \)-open set. Let \( \{A_\alpha : \alpha \in A\} \) be a family of \( \mu^* \)-open sets. Then there exist \( \mu \)-open sets \( U_\alpha \) such that \( U_\alpha \subseteq A_\alpha \subseteq cl(U_\alpha) \) for each \( \alpha \in A \). Thus \( \cup\{U_\alpha : \alpha \in A\} = U \) (say) \( \subseteq \cup\{A_\alpha : \alpha \in A\} \subseteq cl(U) \) where \( U \) is \( \mu \)-open showing that the union of \( \mu^* \)-open sets is \( \mu^* \)-open.

**Example 2.5.** (a) Let \( X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{a, c\}, X\} \) and \( \mu = \{\emptyset, \{c\}, \{a, c\}\} \). Then \( \mu \) is a GT on the topological space \((X, \tau)\). It can be checked easily that \( \{b, c\} \) is a \( \mu^* \)-open set which is not a \( \mu \)-open set.

(b) Let \( X = \{a, b, c\}, \mu = \{\emptyset, \{a, b\}, \{a, c\}, \{b, c\}, X\} \) and \( \tau = \{\emptyset, \{a\}, \{a, b\}, \{a, b\}, X\} \). Then \( \mu \) is a GT on the topological space \((X, \tau)\). It can be easily verified that \( \{a, b\} \) and \( \{a, c\} \) are both \( \mu^* \)-open but their intersection \( \{a\} \) is not so.

**Theorem 2.6.** Let \( \mu \) be a GT on a topological space \((X, \tau)\) and \( A \) be a \( \mu^* \)-open set such that \( A \subseteq B \subseteq cl(A) \). Then \( B \) is also a \( \mu^* \)-open set.

**Proof.** As \( A \) is \( \mu^* \)-open, there exists a \( \mu \)-open set \( U \) such that \( U \subseteq A \subseteq cl(U) \). Thus \( U \subseteq B \). Also \( cl(A) \subseteq cl(U) \Rightarrow B \subseteq cl(U) \). Thus \( U \subseteq B \subseteq cl(U) \). Thus \( B \) is \( \mu^* \)-open.

3. \( \mathcal{I}_\mu \)-open sets

**Definition 3.1.** Let \( \mu \) be a GT on an ideal topological space \((X, \tau, I)\). A subset \( A\) of \( X \) is called \( \mathcal{I}_\mu \)-open if there exists a \( \mu \)-open set \( U \) such that \( U \setminus A \in \mathcal{I} \) and \( A \setminus cl(U) \in \mathcal{I} \).

If \( A \in \mathcal{I} \), then \( A \) is an \( \mathcal{I}_\mu \)-open set and also by Theorem 2.2, every \( \mu^* \)-open set (hence every \( \mu \)-open set) is \( \mathcal{I}_\mu \)-open for any ideal \( \mathcal{I} \) on \( X \).

**Example 3.2.** (a) Let \( X = \{a, b, c\}, \mu = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}, \tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\} \) and \( \mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \). Then \( \mu \) is a GT on the ideal topological space \((X, \tau, \mathcal{I})\). It can be verified that \( \{b\} \) is \( \mathcal{I}_\mu \)-open but not \( \mu^* \)-open.
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