Physica A 390 (2011) 3120–3135

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Replicating financial market dynamics with a simple self-organized critical lattice model

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ARTICLE INFO

Article history:
Received 28 October 2010
Received in revised form 4 February 2011
Available online 22 April 2011

Keywords:
Econophysics
Financial markets
Statistical field theory
Self-organized criticality

ABSTRACT

We explore a simple lattice field model intended to describe statistical properties of high-frequency financial markets. The model is relevant in the cross-disciplinary area of econophysics. Its signature feature is the emergence of a self-organized critical state. This implies scale invariance of the model, without tuning parameters. Prominent results of our simulation are time series of gains, prices, volatility, and gains frequency distributions, which all compare favorably to features of historical market data. Applying a standard GARCH(1,1) fit to the lattice model gives results that are almost indistinguishable from historical NASDAQ data.

Published by Elsevier B.V.

1. Introduction

From a reductionist perspective the statistical physics of a large number of dynamical systems in nature originates from nonlinear processes at the microscopic level. In many cases, this leads to phenomena characterized in the literature by the terms of chaos, complexity, fractal geometry, and criticality. These scenarios are quite ubiquitous, thus not limited to basic physical systems, where turbulence comes to mind for example, but also to applications in biology, geology, social networks, economic systems, and finance, to name a few [1]. The literature on the subject is prodigious.

Our current interest in the subject stems from a recent simulation of financial market dynamics [2]. At the root of that study is a microscopic model based on the principle of gauge invariance, assuming that one of the key mechanisms of trader behavior is independent of any scale (currency unit, for example) used in the market transactions [3]. In technical terms, the model is a quantum field theory based on the gauge group $G = \mathbb{R}^+$, the dilation group, which implies scale invariance of the market model with respect to ordinary multiplication of prices with positive real numbers. The quantum aspect of that model implements the empirical observation that arbitrage opportunities, i.e. realizing a profit via transactions in different markets, vanish quickly because of market dynamics.

At this stage, the model does not provide a mechanism for describing a complex system, as it should, given the empirical evidence. The distribution of market returns, if analyzed appropriately [2], exhibits fat tails (probabilities larger than Gaussian) the likes of which are observed in many high frequency financial markets. However, this is not a consequence of the intrinsic dynamics of the model. In order to remedy this situation, in the present article, we study an abridged model with local interactions that lead to a self-organized complex market model. Although our goal is to eventually combine the gauge model with features of the abridged model discussed here, the latter, despite its simplicity, produces salient characteristics of actual financial markets surprisingly well. Among those are the semblance of return time series, returns frequency distributions, and the nature of volatility. The market volatility, in particular, is a subject of intense research [4–6].

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0378-4371/$ – see front matter. Published by Elsevier B.V.
doi:10.1016/j.physa.2011.04.017

These features are promising enough to study this simple model in its own right, which is the subject of this work. Along a one-dimensional lattice representing discrete time, the field on the sites are interpreted as returns in a model market. An updating algorithm is then applied which is loosely fashioned after the well-known proposal by Bak, Tang and Wiesenfeld (BTW) [7]; see also Refs. [8,9,1,10]. The idea is to develop a market model that is driven by microscopic entities, say traders, such that their interaction leaves the lattice field in a self-organized critical (SOC) state. In a critical state, among other things, long-range correlations of suitable observables lead to power law behavior with respect to scaling transformations. Self-organization means that the system is driven to criticality without fine-tuning any external parameters, i.e. solely by its intrinsic dynamics.

It is generally realized that financial markets, being prime examples of social systems, exhibit SOC [11–15]. However, to the best of our knowledge, attempts to model those from a microscopic point of view are rare [16]. In Refs. [17,18] percolation clusters act as investors. Assigning random percolation probabilities, power law behavior is found for the usual observables derived from stock market prices [19]. Another example of such an attempt close to the BTW evolution model is the work of Bartolozzi et al. [20]. Though close to our work in spirit, significant differences exist in terms of the updating strategy and interpretation. Our implementation of our lattice field model will produce price time series, returns and their distributions, volatility time series and their clustering features that are all strikingly similar to historical market data.

Why would one like to have a market model in the first place? After all there are myriads of historical data being collected every day. A good reason is the fact that all of the collected data are merely instances of a random draw from some probability distribution, just like one throw of dice gives only one result, hiding the statistics behind it. A stochastic model on the other hand will enable us to study any number of market instances, and collect ensembles in the language of statistical physics. Observables, as averages endowed with errors, could be computed. Ultimately, a successful model could provide probability distributions for future prices, and thus be an invaluable tool for risk analysis, and the like.

2. Lattice model

We consider the simplest lattice market model conceivable, a one-dimensional chain of $n+1$ sites with labels $j = 0 \ldots n$, where j indicates discrete time $t = j\Delta$ in steps of some arbitrary unit Δ. The sites are populated with a real-valued field r with components $r_j \in \mathbb{R}$. As it turned out it is essential to interpret the field components r_j directly as investment returns. The returns are defined as

$$r_j = \log(\Phi_j/\Phi_{j-1}) \tag{1}$$

where $\Phi_j = P_j/C$ is the price of an investment instrument, such as a stock or index fund for example, and C is a unit (currency, shares, etc.). The continuum version of (1) can be surmised from taking the limit $\Delta \to 0$ in

$$\log(\Phi(t)/\Phi(t-\Delta)) = \Delta \frac{\mathrm{d}}{\mathrm{d}t} \log \Phi(t) + \mathcal{O}(\Delta^2), \tag{2}$$

where $\Phi(t) = \Phi_j$ at $t = j\Delta$.

If we understand the linear lattice as a stand-alone model, the interpretation of r_j as investment returns is supported, with hindsight, only by the outcome of the simulation. However, there is a rather revealing connection to the gauge field model mentioned in the Introduction [2,3] that lends additional support to this interpretation. The discussion of a single-asset gauge model in Section 5:4 of Ref. [3] is relevant to our case. Fixing the gauge such that cash and the asset are measured in the same unit turns out to be convenient. The curvature field, living on the dual lattice, inherits the arbitrage gains from the plaquettes of the gauge field. In Appendix A, we have adapted those deliberations within the context of Ref. [2], adding a few facets. Within that framework the returns r_j naturally correspond to the arbitrage gains of the gauge model.

In order to endow the field r with dynamics we find inspiration in the popular evolutionary model by Bak and Sneppen [9,1,10]. In that context, the field components are fitness values, say $f_j \in [0, 1]$, assigned to the sites of a lattice. The updating process consists in finding the site j_s with $f_{j_s} = \min\{f_j : j = 0 \ldots n\}$, i.e. the least adapted species. Then f_{j_s} and the values $f_{j_s \pm 1}$ of the two next neighbors are replaced with uniformly distributed random numbers from $[0, 1]$. This prescription, when iterated many times $s = 0, 1 \ldots \infty$, leads to a stationary state of the lattice field where a single perturbation can lead to a burst of activity, called an avalanche. The frequency distribution of avalanche sizes is found to follow a power law. A power law is a signature feature of a critical state. Since no tuning of a model parameter is needed, the phenomenon is known as self-organized criticality (SOC). The model is very robust in the sense that changing the updating prescription, within reasonable bounds, will still lead to SOC. A rigorous discussion, containing analytical results, may be found in Ref. [10].

In the context of the financial market model we adopt a modified version of Bak's updating prescription. We select periodic boundary conditions with period $n+1$, such that $r_{n+1} = r_0$ and $r_{-1} = r_n$. In terms of the returns r_j, we define

$$v_j = r_j(r_{j+1} - r_{j-1}) \tag{3}$$

$$V_j = |v_j| \tag{4}$$

and call

$$V = \max\{V_j : j = 0 \ldots n\} \tag{5}$$