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Memory effect and multifractality of cross-correlations in financial markets

Tian Qiu [a,*], Guang Chen [a], Li-Xin Zhong [b], Xiao-Wei Lei [c]

[a] School of Information Engineering, Nanchang Hangkong University, Nanchang, 330063, China
[b] School of Journalism, Hangzhou Dianzi University, Hangzhou, 310018, China
[c] Department of Physics, Chongqing University of Arts and Sciences, Chongqing 402160, China

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ABSTRACT

We introduce an instantaneous and an average instantaneous cross-correlation function to detect the temporal cross-correlations between individual stocks based on the daily data of the United States and the Chinese stock markets. The memory effect of the instantaneous cross-correlations is investigated by applying the detrended fluctuation analysis (DFA), where the DFA exponents can be partly explained by the correlation function from the common sense. Long-range memory is observed for the average instantaneous cross-correlations, and persists up to a month magnitude of timescale for the United States stock market and half a month magnitude of timescale for the Chinese stock market. In addition, multifractal nature is investigated by a multifractal detrended fluctuation analysis.

1. Introduction

In the past years, dynamics of financial markets has drawn much attention of physicists [1–20]. Based on large amounts of historical data, some stylized facts have been revealed, such as the "fat tail" distribution of the price return, and the so-called "volatility clustering" where the magnitude of returns is observed to be long-range correlated [1–4]. On the other hand, different models and theoretical approaches have been developed to describe financial markets [3,4,21,7,22–25].

From the view of many-body systems, long-range correlation of the volatilities should originate from the strong interactions among individual stocks. To investigate however individual stocks interact with each other, the cross-correlation function is widely adopted as a common mathematical tool. Random and nonrandom properties of the cross-correlations and the relevant economic sectors are revealed [18–20,26–34]. Correlation-based hierarchical or network structures are studied with the graph or complexity theory [35–41]. The so-called pull effect is found with a time-dependent cross-correlation function [42].

Within this framework, dynamics of the cross-correlations has attracted an increasing interest of physicists. Dynamics of the cross-correlation functions and the eigenvalues of the cross-correlation matrices are widely investigated. Recently, a detrended cross-correlation analysis is proposed to investigate the memory effect of the cross-correlations between two time series [43], where the long-range memory is characterized by a power law scaling. The relevant extensive studies and applications are implemented [44–47].

In this paper, we introduce an instantaneous cross-correlation and an average instantaneous cross-correlation by considering correlations of pairs of stocks at a single time step, and therefore can quantify the temporal correlation between individual stocks with the local information. Based on the daily data of the United States and the Chinese stock markets, we

* Corresponding address: 696 South Fenghe Avenue, School of Information Engineering, Nanchang Hangkong University, Nanchang, 330063, China.
E-mail address: tianqiu.edu@gmail.com (T. Qiu).

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study the memory effect of the instantaneous cross-correlations. More importantly, we examine the memory effect of the average instantaneous cross-correlations with different timescales of returns, and reveal the relevant multifractal nature.

The remainder of this paper is organized as follows. In the next section, we give the details of the datasets. In Section 3, we investigate the cross-correlations of pairs of individual stocks from the common sense and introduce the instantaneous cross-correlation, with both the price return and the volatility. Possible relation between the cross-correlation from the common sense and the memory effect of the instantaneous cross-correlation is discussed. In Section 4, we introduce the average instantaneous cross-correlation, and study the corresponding memory effect with different timescales of returns. Section 5 reveals the multifractal properties of the average instantaneous cross-correlations. Finally, Section 6 contains the conclusion.

2. Datasets

To obtain a comprehensive study, we analyze two different databases, the New York Stock Exchange (NYSE) and the Chinese Stock Market (CSM). The two markets are two typical financial markets, which represent the mature and the emerging markets, respectively. The NYSE is one of the oldest stock exchanges, whereas the CSM is a newly set up market in 1990. The investigated stocks of the NYSE include stocks of Basic Materials, Conglomerates, Consumer Goods, Finance, Healthcare, Industrial Goods, Services, Technology, Utilities, and of the CSM include stocks of Finance, Information Technology, Energy Resources, Basic Materials, Daily Consumer Goods, Non-Daily Consumer Goods, Industry, Public Utility, Medical and Health Care. We study the daily data of 249 individual stocks with 2900 data points from the year 1997 to 2008 for the NYSE, and the daily data of 259 individual stocks, with 2633 data points from the year 1997 to 2007 for the CSM.

3. Cross-correlations from the common sense and the memory effect of the instantaneous cross-correlations

We firstly consider the cross-correlation from the common sense, where the correlation function is defined with both the price return and the volatility. The price return r_i of the ith stock is defined as the logarithmic price return of the ith stock over a time interval $\Delta t'$

$$r_i(t') = \ln(y_i(t')) - \ln(y_i(t' - \Delta t')), \tag{1}$$

where $y_i(t')$ is the price of the stock $i = 1, \ldots, N$ at time t', and $\Delta t'$ is the timescale. The volatility v_i of stock i is denoted by the absolute value of the price return, i.e. $v_i(t') = |r_i(t')|$ [27]. To compare different stocks, we normalize the price return and the volatility as

$$R_i(t', \Delta t') = \frac{r_i(t') - \langle r_i(t')\rangle}{\sigma_i^r}, \tag{2}$$

$$V_i(t', \Delta t') = \frac{v_i(t') - \langle v_i(t')\rangle}{\sigma_i^v}, \tag{3}$$

where $\sigma_i^r = \sqrt{\langle r_i^2\rangle - \langle r_i\rangle^2}$ and $\sigma_i^v = \sqrt{\langle v_i^2\rangle - \langle v_i\rangle^2}$ are the standard deviations of r_i and v_i, respectively. The $\langle \cdots \rangle$ takes time average over t'. The Return-based Cross-Correlation (RCC) and the Volatility-based Cross-Correlation (VCC) are then defined as

$$RCC_{ij} = \langle R_i(t')R_j(t')\rangle, \tag{4}$$

$$VCC_{ij} = \langle V_i(t')V_j(t')\rangle. \tag{5}$$

The $\langle \cdots \rangle$ takes time average over t'. The RCC_{ij} and the VCC_{ij} represent the cross-correlation between the returns and the volatilities of stocks i and j, respectively. If $RCC_{ij} > 0$ ($VCC_{ij} > 0$), it indicates that the returns (volatilities) of stocks i and j are positively correlated; if $RCC_{ij} < 0$ ($VCC_{ij} < 0$), it indicates that their returns (volatilities) are anti-correlated; if $RCC_{ij} = 0$ ($VCC_{ij} = 0$), it indicates that their returns (volatilities) are uncorrelated. The RCC and the VCC provide powerful measurements for the static cross-correlation between two individual stocks.

In our study, to examine the dynamic behavior of the cross-correlations, we introduce an Instantaneous Cross-correlation, which is also defined with both the Return (RIC) and the Volatility (VIC),

$$RIC_{ij}(t') = R_i(t')R_j(t'), \tag{6}$$

$$VIC_{ij}(t') = V_i(t')V_j(t'). \tag{7}$$

The RIC_{ij} (VIC_{ij}) considers the cross-correlation between the price returns (volatilities) of stocks i and j at a single time step, and therefore fluctuates according to the price dynamics. The memory effect of the RIC (VIC) series can be detected by the autocorrelation function, which however shows large fluctuations for nonstationary time series. Therefore, we apply the detrended fluctuation analysis (DFA) method [48,49].

For a time series $A(t')$, we eliminate the average trend from the time series by introducing $B(t') = \sum_{t''=1}^{t'}[A(t'') - A_{\text{ave}}]$, where A_{ave} is the average of $A(t')$ in the total time interval $[1, T]$. Dividing the total time interval into windows N_t with a