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An analysis of the effect of noise in a heterogeneous agent financial market model

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ABSTRACT

Heterogeneous agent models (HAMs) in finance and economics are often characterised by high dimensional nonlinear stochastic differential or difference systems. Because of the complexity of the interaction between the nonlinearities and noise, a commonly used, often called indirect, approach to the study of HAMs combines theoretical analysis of the underlying deterministic skeleton with numerical analysis of the stochastic model. However, it is well known that this indirect approach may not properly characterise the nature of the stochastic model. This paper aims to tackle this issue by developing a direct and analytical approach to the analysis of a stochastic model of speculative price dynamics involving two types of agents, fundamentalists and chartists, and the market price equilibria of which can be characterised by the stationary measures of a stochastic dynamical system. Using the stochastic method of averaging and stochastic bifurcation theory, we show that the stochastic model displays behaviour consistent with that of the underlying deterministic model when the time lag in the formation of price trends used by the chartists is far away from zero. However, when this lag approaches zero, such consistency breaks down.

1. Introduction

Traditional economics and finance theory based on the paradigm of the representative agent with rational expectations has not only been questioned because of the strong assumptions of agent homogeneity and rationality, but has also encountered some difficulties in explaining the market anomalies and stylised facts that show up in many empirical studies, including high trading volume, excess volatility, volatility clustering, long-range dependence, skewness, and excess kurtosis (see Pagan, 1996; Lux, 2009 for a description of the various anomalies and stylised facts). As a result, there has been a rapid growth in the literature on heterogeneous agent models that is well summarised in the recent survey papers by Hommes (2006), LeBaron (2006), Hommes and Wagener (2009) and Chiarella et al. (2009). These models characterise the dynamics of financial asset prices and returns resulting from the interaction of heterogeneous agents having different attitudes to risk and having different expectations about the future evolution of prices. For example, Brock and Hommes (1997, 1998) propose a simple *Adaptive Belief System* to model economic and financial markets. A key aspect of these models is that they exhibit feedback of expectations. The resulting dynamical system is *nonlinear* and, as Brock and Hommes (1998) show, capable of generating complex behaviour from local stability to (a)periodic cycles and even chaos.

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By adding noise to the underlying deterministic system and using the simulation approach, many models (see, for example, Hommes, 2002; Chiarella et al., 2006a, 2006b) are able to generate realistic time series. In particular, it has been shown (see for instance Hommes, 2002; He and Li, 2007; Gaunersdorfer et al., 2008; Lux, 2009) that such simple nonlinear adaptive models are capable of capturing important empirically observed features of real financial time series, including fat tails, clustering in volatility and power-law behaviour (in returns). Most of the simple stylised evolutionary adaptive models that one encounters in the literature are analysed within a discrete-time framework and their numerical analysis provides insights into the connection between individual and market behaviour.

One of the most important issues for many heterogeneous agent asset pricing models is the interaction of the behaviour of the heterogeneous agents and the interplay of noise with the underlying nonlinear deterministic market dynamics. Indeed Chiarella et al. (2006b) and He and Li (2007) in their simulations find that these two effects interact in ways which are not yet understood at a theoretical level. The noise can be either fundamental noise or market noise, or both. The commonly used approach (except the stochastic approach developed in Lux, 1995, 1997, 1998), referred to as the *indirect approach* for convenience, is first to consider the corresponding deterministic "skeleton" of the stochastic model where noise terms are set to zero and to investigate the dynamics of this nonlinear deterministic system by using deterministic stability and bifurcation theory; one then uses simulation methods to examine the interplay of various types of noise with the deterministic dynamics. This approach relies on a combination of simulations and faith that the properties of the deterministic system carry over to the stochastic one. However, it is well known that the dynamics of stochastic systems can be very different from the dynamics of the corresponding deterministic systems, see for instance Mao (1997). Ideally we would like to deal directly with the dynamics of the stochastic systems, but this *direct approach* can be difficult.

A number of stochastic asset pricing models have been constructed in the heterogeneous agents literature. The earliest one we are aware of is that of Föllmer (1974) who allows agents' preferences to be random and governed by a law that depends on their interaction with the economic environment. Rheinlaender and Steinkamp (2004) study a one-dimensional continuously randomised version of Zeeman's (1974) model and show a stochastic stabilisation effect and possible sudden trend reversal. Wenzelburger (2004) develops a stochastic version of the Brock and Hommes model. Brock et al. (2005) study the evolution of a discrete financial market model with many types of agents by focusing on the limiting distribution over types of agents. They show that the evolution can be well described by the large type limit (LTL) and that a simple version of LTL buffeted by noise is able to generate important stylised facts, such as volatility clustering and long memory, observed in real financial data. Föllmer et al. (2005) consider a discrete-time financial market model in which adaptive heterogeneous agents form their demands and switch among different expectations stochastically via a learning procedure. They show that, if the probability that an agent will switch to being a "chartist" is not too high, the limiting distribution of the price process exists, is unique and displays fat tails. Other related works include Hens and Schenk-Hoppé (2005) who analyse portfolio selection rules in incomplete markets where the wealth shares of investors are described by a discrete random dynamical system, Böhm and Chiarella (2005) who consider the dynamics of a general explicit random price process of many assets in an economy with overlapping generations of heterogeneous consumers forming optimal portfolios, Böhm and Wenzelburger (2005) who provide a simulation analysis of the empirical performance of portfolios in a competitive financial market with heterogeneous investors and show that the empirical performance measure may be misleading. In particular, by assuming that agent demand is derived from intertemporal optimisation and agents are allowed to switch between strategies, Horst and Wenzelburger (2008) develop a discrete-time stochastic model and show that the limiting distributions may be either unimodal or bimodal, exhibiting a bifurcation-type phenomenon. Most of the cited papers focus on the existence and uniqueness of limiting distributions of discrete time models. For continuous time models, we refer to the one-dimensional continuously randomised version of Zeeman's (1974) stock market model studied by Rheinlaender and Steinkamp (2004) and the work of Horst and Rothe (2008) who examine the impact of time lags in continuous time heterogeneous agent models.

In this paper, we extend the continuous-time deterministic models of speculative price dynamics of Beja and Goldman (1980) and Chiarella (1992) to a stochastic model, of which the market price equilibria can be characterised by stationary measures. We choose this very basic model of fundamentalist and speculative behaviour as it captures in a very simple way the essential aspects of the heterogeneous boundedly rational agents paradigm. Economically, in the agent-based financial market model with stochastic noise, we study how the distributional properties of the model, which can be characterised by the stationary distribution of the market price process, change as agents' behaviour changes and how the market price distribution is influenced by the underlying deterministic dynamics. Mathematically, we seek to understand the connection between different types of attractors and bifurcations of the underlying deterministic skeleton and changes in stationary measures of the stochastic system. By comparing both the *indirect* and *direct* approaches, we examine the consistency of the results under both approaches. We show that the stationary measure of the stochastic model displays a bifurcation of very similar nature to that of the steady state of the underlying deterministic model when the time lag of chartist expectations is far away from zero. Using the stochastic method of averaging, we show through a so-called phenomenological (P)-bifurcation analysis that the stationary measure displays a significant qualitative change near a threshold value from single-peak (unimodal) to crater-like (bimodal) joint distributions (and also marginal distributions) as chartists become more active in the market. However, when the time lag in the formation of price trends used by chartists approaches zero, the stochastic model can display very different features from those of its underlying deterministic model.

The paper unfolds as follows. Section 2 reviews the heterogeneous agents financial market models developed by Beja and Goldman (1980) and Chiarella (1992). Sections 3 and 4 examine the dynamical behaviour of the stochastic model