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Financial market equilibria with cumulative prospect theory

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ABSTRACT

The paper first shows that financial market equilibria need not to exist if agents possess cumulative prospect theory preferences with piecewise-power value functions. This is due to the boundary behavior of the cumulative prospect theory value function, which might cause an infinite short-selling problem. But even when a non-negativity constraint on final wealth is added, non-existence can occur due to the non-convexity of CPT preferences, which might cause discontinuities in the agents' demand functions. This latter observation also implies that concavification arguments which has been used in portfolio allocation problems with CPT preferences do not apply to our general equilibrium setting with finite many agents. Existence of equilibria is established when non-negativity constraints on final wealth are imposed and there is a continuum of agents in the market. However, if the original prospect theory is used instead of cumulative prospect theory, then other discontinuity problems can cause non-existence of market equilibria even in this case.

1. Introduction

The general equilibrium model provides the foundation to most of the theoretical and empirical developments in asset pricing. For example, the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) is a specific general equilibrium model (see, for example, Hens and Pilgrim, 2003) that can be considered "one of the two or three major contributions of academic research to financial managers during the post-war era" (Jagannathan and Wang, 1996).

Asset pricing models in finance usually make specific assumptions on agents' preferences. Expected utility preferences with constant relative or absolute risk aversion are the classical paradigm. And under these assumptions, agents' preferences are convex and the conditions for existence and uniqueness of financial market equilibria can be determined (see Hens and Pilgrim, 2003; Magill and Quinzii, 1996 for an overview). Without convexity, financial market equilibria might not exist if there is a finite number of agents. However, financial markets with a large number of agents where each of them has

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an insignificant impact on trading, can be modeled by a continuum of agents. Under the assumption of a continuum of agents, market equilibria exist under less restrictive conditions on agents' preferences (Aumann, 1966; Schmeidler, 1969; Hildenbrand, 1974; Yamazaki, 1978).

The prospect theory (PT) of Kahneman and Tversky (1979) and the cumulative prospect theory (CPT) of Tversky and Kahneman (1992) summarize several violations of the expected utility theory as observed in laboratory experiments. In (cumulative) prospect theory, utility is defined over gains and losses relative to a reference point and decision makers have different risk preferences over gains and losses with a convex–concave value function. Moreover, (cumulative) prospect theory introduces a probability weighting function to describe decision makers' preferences, i.e., it is assumed that decision makers do not evaluate outcomes according to the true probabilities, but according to decision weights. The probability weighting function plays an important role in (cumulative) prospect theory. For example, it is not true that in (cumulative) prospect theory risk attitudes are only determined by the value function. Convex preferences can arise even if the value function is non-concave. Consequently, it is not clear whether the existence of financial equilibria can be established under general conditions when agents possess CPT preferences.

This paper shows that in a general equilibrium model with a finite number of agents financial market equilibria need not exist if agents possess (heterogenous) CPT preferences. Due to the boundary behavior of the CPT value function, if agents do not face non-negativity constraints on final wealth, non-existence of equilibria is obtained when agents possess *heterogenous* CPT preferences. In this case, it might occur that for any set of prices, at least one agent optimally infinitely short-sells one of the assets. This happens if we apply the classical specification of CPT using a piecewise-power value function. As a consequence, market demand is infinite for at least one of the assets, for which the market clearing condition will never be satisfied and, therefore, a financial market equilibrium does not exist. A similar result is shown by De Giorgi et al. (2003), who use a different setup and impose restrictive assumptions on assets' returns. Note that in general unbounded consumption sets are not sufficient for nonexistence of equilibria, as shown by Werner (1987) and, more recently, by Alloucha et al. (2006). The infinite short-selling problem has also been identified by Jin and Zhou (2008), who in a continuous time setting derive the conditions under which the portfolio selection model with cumulative prospect theory is "well-posed", i.e., the optimal feasible portfolio has finite prospective value. Jin and Zhou (2008) make use of probability weighting to solve the infinite short-selling problem but they do not study financial market equilibria.

In order to avoid infinite short-selling we impose non-negativity constraints on final wealth. However, we show that, while non-negativity constraints on final wealth solve the infinite short-selling problem, non-existence of equilibria still arises. This result is not an obvious consequence of the fact that CPT assumes a convex value function over the domain of losses. Indeed, as we discussed above, probability weighting also enters into the specification of CPT preferences, and it can mitigate the effect of the convex value function. This happens, e.g., in portfolio selection problems as demonstrated by Jin and Zhou (2008), where concavification arguments are used to find optimal asset allocations. However, in our general equilibrium setting, concavification arguments do not apply and CPT preferences appear to be non-convex. This causes discontinuities in the assets' demand function, which also happen if agents have *homogeneous* CPT preferences.

Recently, Xi (2007) has proved the existence of financial market equilibria if agents possess S-shaped value functions. Apparently, this finding contrasts with our examples of non-existence. However, in Xi (2007) existence is obtained only under the condition that discounted portfolio payoffs are strictly larger than the initial wealth, which is taken as reference point. Under this assumption agents never face losses and thus only the concave part of their value function determines the aggregate assets' demand. This is the classical case of expected utility preferences with a concave utility function. Therefore, our examples of non-existence (which consider the most relevant case where both gains and losses occur with strictly positive probability) shows that no general existence results can be obtained if the number of agents is finite.

In order to deal with the non-existence of equilibria due to the non-convexity of CPT preferences we assume that there is a continuum of agents participating in the financial market. We show that CPT preferences satisfy the conditions for existence of equilibria as established by Aumann (1966) and later generalized by Hildenbrand (1974), even when considering probability weighting. By contrast, these conditions are generally violated with PT preferences and existence of financial market equilibria cannot be established in this case even in the presence of a continuum of agents. The reason for this surprising discrepancy between the two models lies in the continuity properties that CPT satisfies, but PT does not. Since PT and CPT only differ by how probability weighting is modeled, this result further demonstrates the importance of probability weighting for the specification of agents' preferences.

The reminder of this paper is structured as follows. In Section 2 we present the model setup. In Section 3 we discuss two examples of financial markets with a finite number of agents with cumulative prospect theory preferences where financial market equilibria do not exist. Section 4 shows that financial market equilibria exist if non-negativity constraints are imposed and there is a continuum of agents. We also provide an example with a continuum of agents and prospect theory preferences, where a financial market equilibrium does not exits. Section 5 concludes. Except for the main theorem in Section 4, all the proofs can be found in Appendix A.

2. The model

We consider a static investment model: in $t = 0$ agents decide their investment strategies, while in $t = 1$ they consume their total portfolio's payoff and endowment. Uncertainty is given by a finite probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\Omega = \{1, \ldots, S\}$, $\mathcal{F} = 2^{|\Omega|}$ is the set of all subsets of Ω, and $\mathbb{P}[\{s\}] = p_s > 0$ is the probability that the state of the nature s appears. There are