Bistable higher harmonics in a hybrid optomechanical system

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1. Introduction

Optomechanics has gained widespread interest, theoretically and experimentally, in the last few years [1], due to their potential applications for precise measurements, such as gravitational waves detection [2] and fine displacement measurements [3]. Optomechanics exploits the coupling between optical modes and mechanical (vibrational) modes (phonon–photon interactions) via radiation pressure. A standard optomechanical system is the Fabry–Perot cavity that consists of one fixed mirror and another movable mirror connected to a fixed wall. The radiation field incident on the mirror propagates through the cavity and produces radiation pressure on the movable mirror. This radiation pressure changes both the cavity length and the resonant frequency of the mechanical resonator and its coupling.

The effect of radiation pressure on the movable mirror induces many effects, e.g., cooling of the mechanical modes and atoms [4–8], amplification of microwave signals [9], photon blockade [10], optical bistability [11], two-mode squeezing photon–phonon interactions [12] and squeezed states coupling in the optomechanical regime [13]. Single photon nonlinearity and nonlinear optomechanics are investigated in [14,15]. In [16], the dynamical behaviour of an optomechanical system was examined where the route from period doubling to chaos was shown in both optical and mechanical modes, which has applications in random number generation, secure communication and chaotic optical sensing [17–19]. Optomechanical systems can be used in effective mass sensing based on the intensity measurement of the output spectrum [20]. Multistability has been reported for optomechanical systems with and without atomic media [21–23]. In the absence of an atomic medium, multistability can be explained as follows: A vibrating mirror element replaces the role of the atomic media to render the reaction of the cavity nonlinearity. The cavity field is deformed by mirror oscillations through the cavity field and the mirror oscillations induce Kerr-like nonlinearity giving rise to a hysteresis cycle of bistability caused by the photon–phonon interaction [24], which is analogous to the usual bistability caused by atomic dipole–field interactions.

When both the atomic medium and oscillating mirror are present in the cavity, there are three interacting elements: the optical cavity mode, the mechanical mode of the mirror and the atomic system. The atoms are coupled to both the mechanical mode and the cavity field [25]. The coupling between the atoms and the mechanical oscillator leads to a new class of optomechanical system [26]. One of the nonlinear phenomena that has emerged from optomechanical systems is optical bistability. The authors in [27] showed that the radiation pressure can be enhanced by considering a two-level atom in the vibrating mirror (as a defect) interacting with the mechanical modes. Furthermore, radiation pressure can be enhanced by the atomic ensemble inside the cavity [22].

Optical bistability for both optical and mechanical fields has been exhibited for hybrid optomechanical systems that include a two-level atom within the movable mirror and a fixed mirror illuminated with two detuned fields [27]. The mechanical resonator is coupled to the cavity field via radiation pressure and the two-level atom via the Jaynes–Cummings interaction. The predicted bistable behaviour in [27] for both the cavity and mechanical field intensities against the driving field is shown at a fixed (nearly ground state) population and with exact resonance between the incident fields. The effect of cross-Kerr coupling nonlinearity on the optomechanical system driven by two coloured
fields was investigated in [28], where the detuning parameter 
between the driving fields changes the behaviour from bistable to tristable.

In this paper, we extend the work in [27] to examine the bistable 
behaviour of the first harmonic components of both the optical and 
mechanical fields in the general case of the off-resonance regime. It is 
worth mentioning that the optical bistable model based on the atomic 
dipole–field interaction beyond the usual rotating wave approximation 
(RWA) [29] shows additional higher harmonic output field components 
simultaneously with the usual bistable (anti-clockwise) hysteresis for 
the fundamental output field component. The first harmonic output 
field component outside the RWA can be further controlled to show 
a one- or two-way switching processes when atomic inhomogeneous 
broadening and transverse input field features are taken into consid-
eration [30]. Possible applications of simultaneous opposite coding of 
both fundamental and first harmonic output field components in optical 
information signal processing were discussed in [29].

The paper is organised as follows: in Section 2, we review the model 
equations for the hybrid optomechanical system. Section 3 presents the 
computational results and Section 4 discusses our conclusions.

2. The model equations

The hybrid optomechanical model – (Fig. 1) – consists of one fixed 
mirror $M_1$ and a movable mirror $M_2$ connected to a spring fixed to a 
wall. A two-level atom, with transition frequency $\omega_0$, and of ground 
and excited states, $|g\rangle$ and $|e\rangle$, respectively, is immersed in the 
movable mirror $M_2$. The single mode cavity field of the frequency $\omega_0$ 
is coupled to a mechanical resonator of frequency $\omega_0$. The mechanical 
resonator interacts with the two-level atom with coupling strength $\kappa$ 
and is quantified by the Jaynes-Cummings Hamiltonian. In addition, 
the interaction between the mechanical resonator and the cavity field is 
of coupling-strength dependent on the radiation-pressure. There is no 
interaction between the atom and the cavity field. Further, a strong 

The nonlinear system of Maxwell–Bloch equations ((1)–(4)) set to zero, 
and using the Fourier decomposition, the analytical expressions for the 
harmonic components ($a_1, b_1, r_1, j_1$); $k = 0, 1, \ldots$ are derived in 
[27] (these expressions are not included here, as they serve no analytical 

In the following section, we present our computational results for 
the bistable behaviour of the first harmonic components of both 
the average photon and phonon numbers $|a_1|^2$ and $|b_1|^2$, of the cavity 
and mechanical modes, respectively, against the driving field $|\Omega|/2\pi$, as 
well against the detuning parameter $\delta$.

3. Computational results

3.1. Input–output switching

The bistable behaviour of the first harmonic components $|a_1|^2$ and 
$|b_1|^2$ of the cavity and mechanical field modes, respectively are shown 
in Figs. 2–3. First, the usual anti-clockwise bistable (2-way switching) 
curves for the fundamental components of the photon numbers $|a_1|^2$ and 
$|b_1|^2$ against the driving field strength $\Omega/2\pi$ [27] are shown in the insets 
of Figs. 2a and 3a.

Fig. 2a, b shows the bistable behaviour of $|a_1|^2$ and $|b_1|^2$ against 
the driving field strength $\Omega$ for the same system parameters as for 
$|a_2|^2$ with $\Delta = 0.2\omega_0$ and $\epsilon = \pi$. The bistable knotted shapes exhibit double 
one-way down-switching at the points $P_1$, $P_2$ for $|a_1|^2$ in Fig. 2a and at 
the points $P_1$, $P_2$ for $|a_2|^2$ in Fig. 2b, simultaneously with the fundamental 
component $|a_1|^2$. Note the difference in the behaviour of $|a_1|^2$ and $|a_2|^2$ 
(Fig. 2a,b): The component $|a_1|^2$ reaches its maximum values $A_1, A_2$ 
in Fig. 2a, while the component $|a_2|^2$ reaches its corresponding maximum 
values $A_1, A_2$ but with $A_1 < A_2$ (Fig. 2b).

The first harmonic component $|b_1|^2$ in (Fig. 3a) shows knotted bistable 
shapes with double down-switching processes in comparison with the usual 
bistable behaviour of $|b_1|^2$. A similar behaviour occurs for the component $|b_2|^2$. For increased $\Delta = 0.3\omega_0$ (Fig. 3b), the two maxima $A_3$ and $A_4$ of the knotted bistable 
shape in Fig. 3a change their locations and structures as follows: The peak at $A_4$ changes to a normal peak $A'_4$, where 
$|b_1|^2$ decays to zero with increasing $|\Omega|$, and the peak $A_4$ converts to 
the usual anti-clockwise bistable behaviour (a similar qualitative change

\begin{align}
\langle a \rangle = a_0 + a_1 e^{i\delta t} + a_2 e^{-i\delta t} = \langle a^+ \rangle^* \\
\langle b \rangle = b_0 + b_1 e^{i\delta t} + b_2 e^{-i\delta t} = \langle b^+ \rangle^* \\
\langle \sigma_+ \rangle = r_0 + r_1 e^{i\delta t} + r_2 e^{-i\delta t} = \langle \sigma_+ \rangle^* \\
\langle \sigma_- \rangle = j_0 + j_1 e^{i\delta t} + j_2 e^{-i\delta t}.
\end{align}
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