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Strategic behavior in financial markets

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ABSTRACT

We describe a financial market as a noncooperative game in strategic form. Agents may borrow or deposit money at a central bank and use the cash available to them in order to purchase a commodity for immediate consumption. They derive positive utility from consumption and from having cash reserves at the end of the day, whereas being bankrupt entails negative utility. The bank fixes interest rates. The existence of Nash equilibria (both mixed and pure) of the ensuing game is proved under various assumptions. In particular, no agent is bankrupt at equilibrium. Asymptotic behavior of replica markets is discussed, and it is shown that given appropriate assumptions, the difference between a strategic player and a price taker is negligible in a large economy.

1. Introduction

The problem of modeling an exchange economy with money and credit as a non-cooperative game has been investigated for more than three decades (see e.g., Shubik and Whitt, 1973 for an early contribution, and Giraud, 2003 for a survey and introduction to a recent issue of the *Journal of Mathematical Economics* devoted exclusively to strategic market games). While this substantial literature led to a deeper understanding of many issues, it is fair to observe that several key difficulties still need to be resolved.

Thus, it would be desirable to have a model for an exchange economy in which (i) there exists money, serving both as a mean of exchange and as a store of value; (ii) agents are price makers (and not just price takers); (iii) there exists a central bank who issues money, accepts deposits, and lends; (iv) bankruptcy is not ruled out, but is penalized. To the best of our knowledge, no model encompassing all these desiderata is available as yet. The early paper (Shubik and Whitt, 1973) does not discuss credit.

Most papers that deal with strategic behavior on financial markets do that in a nonatomic context (of the rather substantial body of literature, let us mention, e.g., Dreze and Polemarchakis, 2005; Dubey and Geanakoplos, 2003; Dubey et al., 2000; Dreze and Polemarchakis, 2001; Goodhart et al., 2004, 2005). This is particularly true for several of the contributions to the above mentioned special issue (see e.g., Dubey and Geanakoplos, 2003; Giraud, 2003; Tsomocos, 2003). A Nash equilibrium of nonatomic games, introduced by Schmeidler (1973) may be regarded (and is viewed so extensively in the literature) as a strategic equilibrium. However, an important difference exists between the finite case and the nonatomic case. In the finite

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case, an action by an individual affects the utility function this individual faces, and not just its argument, whereas in the nonatomic case only the aggregate choices of the agents matter. In the economic setup, agents behave in the continuum case as price takers, rather than as price makers. Thus, while strategic behavior is mentioned in the title of the special issue, an inspection of the models reveals immediately that economic agents behave as if prices were given in advance, and all models could be formulated within the framework of General Equilibrium Theory. Consequently, the equilibrium concept appearing in those contributions may be viewed as Walrasian or competitive, i.e., a price equilibrium. Thus, agents may be regarded as optimizing price takers and do not play strategically a non-cooperative game in the same manner that players in an *n*-person game would. We would not enter here an extensive elucidation of this point for obvious reasons of space and of readers' interest. (There exist in the literature several fairly recent genuinely strategic models of financial markets, such as are mentioned in footnote 1 of Giraud and Stahn, 2008.)

It suffices to note that in the substantial work done in the papers quoted above the nonatomic formulation is prevalent, so that (to our taste) the models do not posses all features that a strategic model should posses. (In such a model the rules should also specify explicitly what happens if many players deviate.)

Let us consider (as an example) the recent study (Geanakoplos et al., 2000). Authors consider a stochastic game with a continuum of players. However, the payoff functions as well as the transition functions of the Markovian game are entwined via the price only. Hence, with fixed prices, the players solve an optimization problem. Now, as there is an atomless player space involved, it turns out that the deviation of a single player does not change the global amount of bids of the other players. The combined effect of these features is the fact that deviating from equilibrium will not improve a players' payoff. Hence the Nash equilibrium constructed is of special nature (a *stationary Markov equilibrium*) which allows every player to follow an *optimization* procedure, given the (equilibrium) price.

Besides, games with a continuum of players (and in particular the above one) have to face a common difficulty: how do players (do the rules of the game) cause an (independent and strategically chosen) set of strategies to become a measurable function? While this is a common feature of many models (e.g., Dubey and Shapley, 1994 have taken great care to justify it), we believe that it may be innocent if players behave in a price taking way but is severe if they act strategically (and Dubey–Shapley themselves note that "others may find in this discussion merely a renewed reason . . . or better, an incentive for seeking alternative resolutions").

We wish to stress that our present model, while close in some sense to the one in Geanakoplos et al. (2000), is really different. We present a noncooperative game with a finite set of players and the Nash equilibrium cannot be regarded as a price equilibrium. Thus, the result of choosing a set of strategies is well defined off the equilibrium and at equilibrium the deviation of a player causes no benefits *despite* the intricate impact of his deviation on bidding and asking for loans.

Analysis of such an equilibrium as suggested here is not entirely obvious (even though the existence proof is based – as usual – on a fixed point argument). In particular, the method of 'backwards induction' which allows for a temporal extension of the equilibrium concept, is not well defined here. The proper concept is that of subgame perfectness which (other than, say, in Geanakoplos et al., 2000) cannot be pursued by standard dynamic programming procedures.

This (somewhat detailed) discussion of the literature should convince the reader that a model including the elements (i) through (iv) described above is still desirable and has *not* been treated so far.

In the model proposed here we consider a finite set of agents engaged in trade of a Shapley–Shubik type (Shapley, 1976; Shapley and Shubik, 1977), and a central bank which may issue money, distribute it as loans, and accept deposits. The central bank has the authority to determine the various interest rates. Agents would derive a negative utility from being bankrupt, whereas positive cash holdings at the end of the period have positive utility, the latter presumably deriving from subsequent use of money at a later period. We suggest here a one-shot model; we plan to construct a multi-period extension in subsequent work. We tried to keep the model as simple as possible, considering only one commodity and imposing "sell-all" restriction (for motivation for this assumption, see e.g., Karatzas et al., 2001, p. 300, footnote 7), so as to facilitate concentrating on the relevant issues.

Our model is described in detail in Section 2. Each agent is endowed with positive amounts of a consumer nondurable commodity and money. Agents issue *b* ids in terms of money towards purchasing a quantity of the consumption good. (Agents cannot consume directly their commodity endowment in whole or parts.) Agents may exceed their endowment (and thus take a loan from the bank), or else they may bid less than their endowment, their money surplus going to the bank as a deposit. There is a central bank in the market which controls the interest rates for deposits and loans and increases the total amount of money, if the books cannot be balanced otherwise. As soon as bids are announced, the price of the commodity is given by Eq. (2.5) as the ratio of the aggregate bid to the aggregate supply of the good. Each agent then receives for consumption the good bought by his bid and the money proceeds of the selling of his commodity endowment. In addition, our agent receives returns from her bank deposit or has to pay the loan (with interest).

At the end of the day, each agent has (1) consumed an amount of the commodity (deriving from it a positive amount of utility), (2) is unable to repay his loan with the prescribed interest, so that he is bankrupt and derives a negative utility from this fact, or else (3) has a positive amount of cash left, from which she derives positive utility. These three components of the total utility are given in (2.3) of Definition 2.1. Besides the usual monotonicity and concavity assumptions we also require that the penalty for bankruptcy be sufficiently large so as to offset the positive utility of high consumption, and impose on the utility functions several technical assumptions in order to facilitate certain existence proofs. We do not try to optimize the bankruptcy rule. We trace the flow of cash in the economy and show that the bank never has to withdraw funds out of the economy. The bank announces a policy concerning interest rates on deposits and loans. Formally, this policy is a (vector-