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A social welfare function characterizing competitive equilibria of incomplete financial markets

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ABSTRACT

A classic characterization of competitive equilibria views them as feasible allocations maximizing a weighted sum of utilities. It has been applied to establish fundamental properties of the equilibrium notion, such as existence, determinacy, and computability. However, it fails for economies with missing financial markets.

We give such a characterization for economies with a single commodity and missing financial markets, by an amended social welfare function. Its parameters capture both the relative importance of households welfare, through the classic welfare weights, as well as the disagreements among them as to the value of the missing markets.

As a by-product, we identify the dimension of the set of interior equilibrium allocations.

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1. Introduction

If no financial markets are missing, following Lange (1942) and Allais (1943), interior allocations of given resources r are competitive equilibria if and only if they solve the program $\max_{\Sigma x^h = r} W_\delta(x)$ for some strictly positive δ, with W_δ being the social welfare function[1]

$$W_\delta(x) := \Sigma \delta^h u^h(x^h). \quad (1)$$

The parameters δ in Lange's social welfare function capture the relative importance of households' welfare. This characterization has been applied to establish fundamental properties of the equilibrium notion: existence, Negishi (1960) and Bewley (1969), determinacy with infinitely long living households, Kehoe and Levine (1985), and computability, Mantel (1971).

If some financial markets are missing as in Radner (1972), however, this equivalence fails: some interior competitive equilibria need not solve the program $\max_{\Sigma x^h = r} W_\delta(x)$ for any strictly positive δ. Moreover, no natural social welfare function W has been found that would rescue this implication.

We extend the characterization to economies with some missing financial markets, by amending the social welfare function. Thus interior allocations of given resources are competitive equilibria if and only if they solve the program $\max_{\Sigma x^h = r} W_{\delta,\mu}(x)$ for some parameters $\delta \in \mathbb{D}$, $\mu \in \mathbb{M}$ living in certain spaces, with $W_{\delta,\mu}$ being the social welfare function

$$W(x) := \Sigma \delta^h u^h(x^h) - \Sigma \mu^h \cdot x^h_{\mathbf{1}}. \quad (2)$$

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[1] Lange characterizes *Pareto optima* in this way. So the above characterization follows from the two welfare theorems. (Lange (1942) is aware of the first one, while Allais (1943) is among the first to rigourously prove the second one.)

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Here, the *social evaluation* of allocations is described by the usual weights δ on households' welfare, and by new charges μ on their future consumption. The parameter δ is interpreted classically, whereas μ is interpreted as the "disagreement" among households as to the "value" of the "missing financial markets", as justified below.

Why does it fail, the equivalence of competitive equilibria and maxima of (1), if some financial markets are missing? On the one hand, any allocation x that maximizes this is Pareto efficient. Indeed, if y were Pareto superior to x, i.e. $(u^h(y^h)) > (u^h(x^h))$, then $W_\delta(y) > W_\delta(x)$ for any $\delta \gg 0$, so x could not be a maximum for any $\delta \gg 0$. On the other hand, some allocations x that are competitive equilibria of incomplete financial markets are Pareto inefficient. Indeed, for almost every initial allocation, every competitive equilibrium allocation is Pareto inefficient; for an exposition of this well-known fact, see Magill and Quinzii (1996).[2] So some competitive equilibria fail to maximize (1) for any $\delta \gg 0$.

We explain in what sense the parameter μ is the "disagreement" among households as to the "value" of the "missing financial markets", by clarifying each of these terms. By "missing financial markets" we mean the orthogonal complement $a^\perp$ of the span of the existing financial instruments a. By "value" of the missing financial markets we mean a linear functional $v : a^\perp \to \mathbb{R}$. The separating hyperplane theorem implies that any linear functional on a finite-dimensional vector space can be represented uniquely as the inner product against a unique element of the vector space—call this element $\hat{v} \in a^\perp$, so that $v(m) = m \cdot \hat{v}$. If each household h thinks such a *value* v^h, the *disagreement* is then the differences from the mean, $\mu^h := \hat{v}^h - mean(\hat{v}^1, \ldots, \hat{v}^H)$. When so defined, the disagreement $\mu = (\mu^h)$ satisfies two properties: (i) $\mu^h \in a^\perp$, because it is a linear combination of points $\hat{v}^h \in a^\perp$ in a vector space, and (ii) $\Sigma\mu^h = 0$, because these are differences from the mean. In sum, imagining that each household has its own v^h, an opinion as to the value of the missing financial markets, then this is the sense of the new parameter in our social welfare function (1)—a matrix $\mu = (\mu^h)$ satisfying conditions (i), (ii).

Our main contribution is a fine characterization of the set $\mathbb{X}$ of interior competitive equilibrium allocations of all the economies parameterized by initial endowment distributions $(e^h)_h$ of given state-contingent, aggregate resources r. In doing so, besides r, we take as given smooth preferences u as in Debreu (1972, 1976), and the asset structure formed by finitely many financial instruments a. The characterization is accomplished in steps, establishing three results.

The **first result** (Theorem 1) is that an allocation $x \gg 0$ is an equilibrium allocation if and only if it solves the program $\max_{\Sigma x^h = r} W_{\delta,\mu}(x)$ for some $(\delta, \mu) \in \mathbb{D} \times \mathbb{M}$, where

$$\mathbb{D} := \left\{ \delta \in \mathbb{R}^H \mid \delta \gg 0, \, \Sigma \frac{1}{\delta^h} = 1 \right\} \qquad \mathbb{M} := \left\{ \mu \in \left(a^\perp\right)^H \mid \Sigma\mu^h = 0 \right\}.$$

We see that the "welfare" parameter δ is normalized in a standard way, and the "disagreement" parameter μ reflects properties (i) and (ii) above.

The **second result** (Proposition 1, part A) identifies the (δ, μ) from the equilibrium allocation as being

$$\delta^h(x) = \frac{1}{D_{x_0} u^h(x^h)} \tag{3}$$

$$\mu^h(x) = \hat{v}^h - mean(\hat{v}^1, \ldots, \hat{v}^H) \quad \text{with } \hat{v}^h := \frac{D_{\mathbf{x_1}} u^h(x^h)}{D_{x_0} u^h(x^h)}.$$

Thus δ^h is the inverse of the marginal utility of present consumption, as usual, and μ is, as interpreted above, the disagreement among households as to the value of the missing financial markets, where each household's "value" $\hat{v}^h$ is concretized as the marginal rates at which it substitutes consumption in future states for consumption in the present state. Here, the abstract notion of "value" as a linear functional $v : a^\perp \to \mathbb{R}$ is made concrete by the idea of marginal willingness to pay as $\Delta \mapsto \Delta \cdot MRS$, the inner product of the infinitesimal change Δ in future consumption against the marginal rates of substitution MRS.

The **third result** refines the first two. Theorem 2 establishes that the relation $x \leftrightarrow (\delta, \mu)$ between $\mathbb{X}$ and $\mathbb{D} \times \mathbb{M}$ is a bijection, smooth in both directions. This implies immediately that the dimension of $\mathbb{X}$ equals the dimension of $\mathbb{D} \times \mathbb{M}$, which is easily shown to be $(H - 1)(1 + m)$, where m is the number of missing financial markets. This nests a well-known fact about complete markets, where $m = 0$: the interior Pareto optima (which are $\mathbb{X}$ by the two welfare theorems) have dimension $H - 1$; see proof 5.2.4 in Balasko (1988).

We focus attention on an exchange economy that has a single good per state and in which asset payoffs are denominated in the numéraire. Although restrictive, this context is interesting both theoretically and for financial applications. Theoretically, because it allows one to concentrate on financial markets, leading aside issues concerning spot markets. As for applications, a single good suffices to embody, in a general equilibrium model, classical models of asset pricing such as the CAPM.

Extensions of our characterization to economies with multiple goods and spot markets are substantially available in two other works, Siconolfi and Villanacci (1991) and Tirelli (2008), whose understanding of the geometry of the equilibrium set is instrumental, respectively, in the study of the indeterminacy and of the welfare properties of equilibria also in the sense of

[2] If there are multiple goods and enough missing financial markets, even the equilibrium use of the existing financial markets is generically Pareto inefficient, as shown by Geanakoplos and Polemarchakis (1986), who pioneer the application of transversality to equilibrium welfare. The intuition for this is due to Stiglitz (1982).

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