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A reflexive toy-model for financial market

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ABSTRACT

We propose a reflexive toy model for market dynamics, based on the idea that existing reflexive loops are generated by the conviction, shared by many market operators, that a certain price follows a certain model. Their trading behaviour will therefore increase the probability that the model predictions are in fact fulfilled. We analytically write the equations generating a reflexive loop stemming from a simple linear regression model, and we show that the resulting toy model yields a peculiar intermittent behavior. The presence of two unstable fixed points is apparent from our numerical calculation and the residence-time distribution density in these points asymptotically follows an inverse-power-law tail. The exponent of this tail, as well as the scaling properties of the model output, are close to those stemming from real-price time series.

1. Introduction

Since its first publication in Refs. [1,2], the so called Soros' theory of reflexivity of financial market has triggered several debates. The fact that G. Soros presented it more from a philosophical point of view did not probably help towards the theory being accepted in the academic world. Anyway, as some recent works show [3,4], a possible mathematical framework can be applied to Soros' ideas in order to evince the behavior of a model based on his view of market dynamics. In a few words, reflexive theory deals with systems where the fact that people are convinced that the dynamics will obey a particular model influences the same behavior of the system, increasing the probability that the model predictions will be fulfilled.

To formalize this idea one defines the cognitive function $y = f(x)$ as the result of the observation of system variables x performed by the observer. The observer is driven in his choices by the value of the variable y, but his behavior at the same time influences the system through the manipulative function $\Phi(y)$. The authors of Refs. [3,4] study the evolution of the system

$$\begin{cases} y(t) = f(x(t)) \\ x(t+1) = \Phi(y(t)). \end{cases} \tag{1}$$

In this case, the dynamics of the system are completely determined by the reflexive feedback. The authors of Refs. [3,4] focus their attention on the presence and stability of fixed points of the system of Eq. (1). They studied the problem in a very general way and did not provide any explicit expressions for these functions.

In this work we follow a quite different approach. We suppose that the evolution of the system variables is due to the reflexive effect and to an intrinsic dynamic behavior. We thus study a system given by

$$\begin{cases} y(t) = f(x(t)) \\ x(t+1) = g(x(t)) + \Phi(y(t), x(t)). \end{cases} \tag{2}$$

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If $\Phi(y(t), x(t)) = 0$ this system reduces to a simple dynamical system driven by the function $g(x)$, without the reflexive effect. In this paper we provide an explicit expression for both $f(x)$ and $\Phi(y(t), x(t))$, obtaining the toy-model for financial market. Our toy model assumes that operators believe in a simplified version of the so called "technical analysis". The attempt of the market operator to find trends in prices behavior constitutes the cognitive function, while the operators' confidence in their technical analysis model, together with the consequence of the resulting trades on the prices, is described by the manipulative function $\Phi(y(t), x(t))$.

We observe that the suggested toy-model presents only two unstable equilibrium states. These states are meta-stable in the sense that the system stays close to these positions for very long residence times. We show that the probability density function of the residence times in these states presents power-law tails, i.e. it is characterized by a power-law asymptotic behavior for large residence times. In the numerical simulations of the model we observe a power-law tail with exponent $\mu \simeq 2$. This means that, even if unstable, the presence of these states deeply influences the dynamics of the model.

We will show that the power law tail exponent and the results obtained on the model with an intermittent time-series-analysis technique, called diffusion entropy [5], are practically the same obtained, with the same techniques, on real market data as shown in Refs. [6,7]. This fact confirms that our toy-model shares some important property in common with real market dynamics.

The outline of the paper is as follows. In Section 2 we present and describe the toy-model and its qualitative behavior. In Section 3 we calculate the probability density function of the waiting times in the unstable equilibrium states. In Section 4 we briefly introduce the diffusion-entropy technique and we compare the results obtained on the model output with those already presented in literature on real data. In Section 5 we draw some conclusions on the real market and suggest further developments stemming from our approach.

2. The reflexive correlation model

The model is based on the following ansatz: suppose that an observer is trying to understand if a model can fit the last N observations, $r(t-\tau)$, with $0 \le \tau \le N-1$, of a dynamical system. The easiest guess consists of a linear model

$$\tilde{r}(t+1) = f(t)r(t), \tag{3}$$

where $\tilde{r}(t+1)$ is the prediction of the model at time $t+1$. Minimizing the error of the model (3), defined as

$$\chi^2 = \sum_{\tau=0}^{N-1} \left[\tilde{r}(t-\tau) - r(t-\tau)\right]^2 = \sum_{\tau=0}^{N-1} \left[f(t)r(t-\tau-1) - r(t-\tau)\right]^2, \tag{4}$$

as a function of $f(t)$, we obtain

$$f(t) = \frac{\sum_{\tau=0}^{N-1} r(t-\tau)r(t-\tau-1)}{\sum_{\tau=0}^{N-1} r(t-\tau-1)^2}. \tag{5}$$

Considering the limit $N \to \infty$, we obtain

$$f(t) = \frac{\langle r(t-\tau)r(t-\tau-1)\rangle}{\langle r(t-\tau)^2\rangle}, \tag{6}$$

where $\langle g\rangle \equiv \lim_{N\to\infty} 1/N \sum_{\tau=1}^{N} g(\tau)$. We thus obtain that the best choice for $f(t)$ consists of the normalized correlation of the previous observations, which, in turns, fulfills the condition

$$-1 \le f(t) \le 1. \tag{7}$$

We now suppose that the earlier proposed fitting procedure is not to be applied to the total series $r(t)$, but only on the last N values; nevertheless, we want $f(t)$ to fulfill the condition (7). We are therefore led to define

$$f(t) = \frac{\frac{1}{N-1}\sum_{\tau=0}^{N-2} r(t-\tau)r(t-\tau-1)}{\frac{1}{N}\sum_{\tau=0}^{N-1} r(t-\tau)^2}. \tag{8}$$

Within our toy model, we make the simplifying hypothesis that N is identical for all the market operators. The function $f(t)$ is therefore the main ingredient of the model defined in Eq. (3). In the case where $r(t)$'s are completely random with a zero mean, we have that $f(t) \simeq 0$. This happens because the autocorrelation of a sequence of N independent identically distributed (i.i.d.) random numbers is approximately equal to zero even for low values of N.