



# A long-range memory stochastic model of the return in financial markets

V. Gontis\*, J. Ruseckas, A. Kononovičius

*Institute of Theoretical Physics and Astronomy of Vilnius University, A. Goštauto 12, LT-01108 Vilnius, Lithuania*

## ARTICLE INFO

### Article history:

Received 20 April 2009

Received in revised form 25 July 2009

Available online 10 September 2009

### PACS:

89.65.Gh

02.50.Ey

05.10.Gg

### Keywords:

Models of financial markets

Stochastic equations

Power law distributions

Long memory processes

## ABSTRACT

We present a nonlinear stochastic differential equation (SDE) which mimics the probability density function (PDF) of the return and the power spectrum of the absolute return in financial markets. Absolute return as a measure of market volatility is considered in the proposed model as a long-range memory stochastic variable. The SDE is obtained from the analogy with an earlier proposed model of trading activity in the financial markets and generalized within the nonextensive statistical mechanics framework. The proposed stochastic model generates time series of the return with two power law statistics, i.e., the PDF and the power spectral density, reproducing the empirical data for the one-minute trading return in the NYSE.

© 2009 Elsevier B.V. All rights reserved.

## 1. Introduction

High frequency time series of financial data exhibit sophisticated statistical properties. What is the most striking is that many of these anomalous properties appear to be universal. Vast amounts of historical stock price data around the world have helped to establish a variety of so-called stylized facts [1–6], which can be seen as statistical signatures of financial processes. The findings as regards the PDF of the return and other financial variables are successfully generalized within a nonextensive statistical framework [7]. The return has a distribution that is very well fitted by  $q$ -Gaussians, only slowly becoming Gaussian as the time scale approaches months, years and longer time horizons. Another interesting statistic which can be modeled within the nonextensive framework is the distribution of volumes, defined as the number of shares traded.

Interesting stochastic models related to the nonextensive statistics include an ARCH process with random noise distributed according to a  $q$ -Gaussian as well as some state-dependent additive–multiplicative processes [8]. These models do capture the distribution of returns, but not necessarily the empirical temporal dynamics and correlations. Additive–multiplicative stochastic models of the financial mean-reverting processes provide a rich spectrum of shapes for the probability distribution function (PDF) depending on the model parameters [9]. Such stochastic processes model the empirical PDF's of volatility, volume and price returns with success when the appropriate fitting parameters are selected. Many other fits are also proposed, including exponential ones [10] applicable for larger time scales.

Nevertheless, there is a necessity to select the most appropriate stochastic models, able to describe volatility as well as other variables in dynamical aspects and long-range correlation aspects.

There is empirical evidence that trading activity, trading volume, and volatility are stochastic variables with the long-range correlation [11–13] and this key aspect is not accounted for in some widely used models. ARCH-like, multiscale models

\* Corresponding author. Tel.: +370 698 12384; fax: +370 5 2784081.

E-mail address: [gontis@itpa.lt](mailto:gontis@itpa.lt) (V. Gontis).

URL: <http://www.itpa.lt/~gontis> (V. Gontis).

of volatility, which assume that the volatility is governed by the observed past price changes over different time scales, have been recently proposed [14,15]. Trading volume and trading activity are positively correlated with market volatility. Moreover, trading volume and volatility show the same type of long memory behavior [16].

Recently we investigated analytically and numerically the properties of stochastic multiplicative point processes [17,18], derived a formula for the power spectrum and related the model with the general form of the multiplicative stochastic differential equation [19,20]. The extensive empirical analysis of the financial market data, supporting the idea that the long-range volatility correlations arise from trading activity, provides valuable background for further development of the long-ranged memory stochastic models [12,13]. The power law behavior of the autoregressive conditional duration process [21] based on the random multiplicative process and its special case the self-modulation process [22], exhibiting  $1/f$  fluctuations, supported the idea of stochastic modeling with a power law PDF and long memory. A stochastic model of trading activity based on an SDE driven Poisson-like process has been already presented in [23]. We further develop an approach of modulating the SDE with a closer connection to the nonextensive statistics in order to model the dynamics of the return in this paper.

Long memory (long-term dependence) has been defined in time domain in terms of autocorrelation power law decay, or in frequency domains in terms of power law growth of low frequency spectra. Despite statistical methodology being developed for data with a long-range dependence and the solid mathematical foundations of the area [24], let us consider behavior of the financial variables only in the frequency domain, analyzing the power spectral density.

In Section 2 we present the nonlinear SDE generating a signal with a  $q$ -Gaussian PDF and power law spectral density. In Section 3 we analyze the tick by tick empirical data for trades on the NYSE for 24 shares and adjust the parameters of the proposed equations to the empirical data. A short discussion and conclusions are presented in Section 4.

## 2. The stochastic model with a $q$ -Gaussian PDF and long memory

Earlier we investigated stochastic processes with long-range memory properties. Starting from the stochastic point process model, which reproduced a variety of self-affine time series exhibiting the power spectral density  $S(f) \sim 1/f^\beta$  scaling as power  $\beta$  of the frequency  $f$  [18], later we introduced a Poisson-like process driven by the stochastic differential equation. The latter served as an appropriate model of trading activity in the financial markets [23]. In this section we generalize an earlier proposed nonlinear SDE within the nonextensive statistical mechanics framework to reproduce the long-range memory statistics with a  $q$ -Gaussian PDF. The  $q$ -Gaussian PDF of stochastic variable  $r$  with variance  $\sigma_q^2$  can be written as

$$P(r) = A_q \exp_q \left( -\frac{r^2}{(3-q)\sigma_q^2} \right), \tag{1}$$

where  $A_q$  is a constant of normalization and  $q$  defines the power law part of the distribution.  $P(r)$  is introduced through the variational principle applied to the generalized entropy [8]

$$S_q = k \frac{1 - \int [p(r)]^q dr}{1 - q}.$$

Here the  $q$ -exponential of variable  $x$  is defined as

$$\exp_q(x) = (1 + (1 - q)x)^{\frac{1}{1-q}} \tag{2}$$

and we assume that the  $q$ -mean  $\mu_q = 0$ . With some transformation of parameters  $\sigma_q$  and  $q$

$$\lambda = \frac{2}{q - 1}, \quad r_0 = \sigma_q \sqrt{\frac{3 - q}{q - 1}}$$

we can rewrite the  $q$ -Gaussian in a more transparent form:

$$P_{r_0,\lambda}(r) = \frac{\Gamma(\lambda/2)}{\sqrt{\pi} r_0 \Gamma(\lambda/2 - 1/2)} \left( \frac{r_0^2}{r_0^2 + r^2} \right)^{\frac{\lambda}{2}}. \tag{3}$$

Looking for the appropriate form of the SDE we start from the general case of a multiplicative equation in the Ito convention with Wiener process  $W$ :

$$dr = a(r)dt + b(r)dW. \tag{4}$$

If the stationary distribution of SDE (4) is the  $q$ -Gaussian (3), then the coefficients of SDE are related as follows [25]:

$$a(r) = -\frac{\lambda}{2} \frac{r}{r_0^2 + r^2} b(r)^2 + b(r) \frac{db(r)}{dr}. \tag{5}$$

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات