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Market stability switches in a continuous-time financial market with heterogeneous beliefs ☆

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ABSTRACT

By considering a financial market of fundamentalists and trend followers in which the price trend of trend followers is formed as a weighted average of historical prices, we establish a continuous-time financial market model with time delay and examine the impact of time delay on market price dynamics. Conditions for the stability of the fundamental price in terms of agents' behavior parameters and time delay are obtained. In particular, it is found that an increase in time delay can not only destabilize the market price but also stabilize an otherwise unstable market price, leading to stability switching as delay increases. These interesting phenomena shed new light in understanding of mechanism on the market stability. When the fundamental price becomes unstable through Hopf bifurcations, sufficient conditions on the stability and global existence of the periodic solution are obtained.

1. Introduction

Technical analysts or "chartists", who use various technical trading rules such as moving averages, attempt to forecast future prices by the study of patterns of past prices and other summary statistics about security trading. Basically, they believe that shifts in supply and demand can be detected in charts of market movements. Despite the efficient market hypothesis of financial markets in the academic finance literature (see Fama, 1970), the use of technical trading rules, such as moving average rules, still seems to be widespread amongst financial market practitioners (see Allen and Taylor, 1990; Taylor and Allen, 1992). This motivates recent studies on the impact of chartists on the market price behavior. Over the last two decades, heterogeneous agent models (HAMs) have been developed to explain various market phenomena and, as the main tool, the stability and bifurcation analysis has been widely used in HAMs. By incorporating heterogeneity and behavior of chartists and examining underlying deterministic models, HAMs have successfully explained the complicated role of chartists in market price behavior, market booms and crashes, and deviations of the market price from the fundamental price. Numerical simulations of the stochastic model based on the analytical analysis of the underlying deterministic model show some potentials of HAMs in generating the stylized facts (such as skewness, kurtosis, volatility clustering and fat tails of returns), and various power laws (such as the long memory in return volatility) observed in financial markets. We refer the reader to Hommes (2006), LeBaron (2006) and Chiarella, Dieci, and He (2009) for surveys of the recent developments in this literature.

Most of the HAMs in the literature are in discrete-time rather than continuous-time setup. To examine the role of moving average rules in market stability theoretically, Chiarella, He, and Hommes (2006) recently propose a discrete-time HAM in which demand for traded assets has both a fundamentalist and a chartist components. The chartist demand is governed by the difference between the current price and a moving average (MA). They show analytically and numerically that an increase in the lag length used in moving average can destabilize the market, leading to cyclic behavior of the market price around the fundamental price. The discrete-time setup facilities economic understanding and mathematical analysis, but it also faces some limitations when expectations of agents are formed in historical prices over different time periods. In particular, when dealing with MA rules in Chiarella, He, and Hommes (2006), different lag lengths used in the MA rules lead to different dimensions of the system which need to be dealt with separately. Very often, an analytical analysis is difficult when the dimension of the system is high. To overcome this difficulty, this paper extends the heterogeneous agent model of the financial market in Chiarella, He, and Hommes (2006) from the discrete-time to a continuous-time framework. The financial market is consisting of a group of fundamentalists and a group of trend followers who use a weighted average of historical prices as price trend. The fundamentalists are assumed to buy (sell) the stock when its price is below (above) the fundamental price. The trend followers

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are assumed to react to buy-sell signals generated by the difference between the current price and the price trend. The model is described mathematically by a system of delay differential equations, which provides a systematic analysis on various moving average rules used in the discrete-time model in Chiarella, He, and Hommes (2006).

Development of deterministic delay differential equation models to characterize fluctuation of commodity prices and cyclic economic behavior has a long history, see, for example, Haldane (1932), Kalecki (1935), Goodwin (1951), Larson (1964), Howroyd and Russell (1984) Mackey (1989). The development further leads to the studies on the effect of policy lag on macroeconomic stability, see for example, Phillips (1954, 1957), Asada and Semmler (1995), Asada and Yoshida (2001) and Yoshida and Asada (2007). In particular, as indicated in Manfredi and Fanti (2004), an important class of delay economic models is that of distributed delay systems governed by Erlangian kernels, which are reducible to higher dimensional ordinary differential equation systems.

Though there is a growing study on various market behavior, in our knowledge, using delay differential equations to model financial market behavior is relatively new. This paper aims to extend Chiarella, He, and Hommes (2006) model in discrete-time to continuous-time with a time delay framework. This extension provides a uniform treatment on the moving average rules with different window length in discrete-time model. Different from the distributed delay of Erlangian kernel type used in economic modelling literature, the delay introduced in this paper is not 'reducible' in general. By focusing on the impact of the behavior of heterogeneous agents, the stabilizing role of the time delay is examined. Sufficient conditions for the stability of the fundamental price in terms of agents' behavior parameters and time delay are derived. Consistent with the results obtained in the discrete-time model in Chiarella, He, and Hommes (2006), it is found that an increase in time delay can destabilize the market price, resulting in oscillatory market price characterized by a Hopf bifurcation. However, in contrast to the discrete-time model, it is also found that, depending on the behavior of the fundamentalists and trend followers, an increase in the time delay can also stabilize an otherwise unstable market price and such stability switching can happen many times. The stability switching is a very interesting and new phenomenon on price dynamics of the HAMs. The stabilizing role of reducible distributed delay has been observed in economic modelling (see Manfredi and Fanti, 2004) and it is of interest to ascertain that this stability is preserved under non-reducible delay introduced in this paper. When the fundamental steady state becomes unstable, the market price displays cyclic behavior around the fundamental price characterized by Hopf bifurcations. We also examine the stability of the Hopf bifurcation and furthermore the global existence of periodic solutions bifurcating from the Hopf bifurcations.

The paper is organized as follows. We first introduce a deterministic HAM with two types of heterogeneous agents in a continuous time framework with time delay in Section 2. In Section 3, we first conduct a stability and bifurcation analysis of the delay differential equation model and then examine the stability of the periodic solution characterized by the Hopf bifurcation. In addition, we obtain some results on the global existence of periodic solutions resulting from the Hopf bifurcation. Section 4 concludes the paper. All the proofs of technical results are given in the appendices.

2. A financial market model with delay

Following the current HAM framework, see for example, Brock and Hommes (1998), Chiarella and He (2002, 2003), and, in particular, Chiarella, He, and Hommes (2006) in discrete-time setup, this section proposes an asset pricing model in a continuous-time framework with two different types of heterogeneous traders, fundamentalists and trend followers, who trade according to fundamental analysis and technical analysis, respectively. The market price is arrived at via a market maker scenario in line with Beja and Goldman (1980), Day and Huang (1990) and Chiarella and He (2003).

Consider a market with a risky asset (such as stock market index) and let $P(t)$ denote the (cum dividend) price per share of the risky asset at time t. To focus on price dynamics, we follow Beja and Goldman (1980) and Day and Huang (1990) and motivate the demand functions of the two different types of traders by their trading rules directly, rather than deriving the demand functions from utility maximization of their portfolio investments with both risky and risk-free assets (as for example in Brock and Hommes, 1998; Chiarella and He, 2003). The market population fractions[1] of fundamentalists and chartists are respectively α and $1-\alpha$, where $\alpha\in[0,1]$.

The fundamentalists trade based on their estimated fundamental price. They believe that the market price $P(t)$ is mean-reverting to the fundamental price $F(t)$ which is assumed to be a constant $F(t)=F$ for simplicity. We assume that the demand of the fundamentalists, $D_f(t)$ at time t, is proportional to the price deviation from the fundamental price, namely,

$$D_f(t) = \beta_f[F-P(t)], \tag{2.1}$$

where $\beta_f>0$ is a constant parameter, measuring the mean-reverting of the market price to the fundamental price, which may be weighted by the risk aversion coefficient of the fundamentalists.

The chartists trade based on charting signals generated from historical prices. Given the well documented momentum trading strategy in empirical literature, see for example Hirshleifer (2001), we assume that the chartists are trend followers. They believe that the future market price follows a price trend $u(t)$. When the current price is above the trend, the trend followers believe the price will rise and they like to take a long position of the risky asset; otherwise, the trend followers will take a short position. We therefore assume that the demand of the chartists is given by

$$D_c(t) = g(P(t)-u(t)), \tag{2.2}$$

where the demand function g satisfies:

$$g'(x)>0,\ g'(0) = \beta_c>0,\ xg''(x)<0 \ \text{ for } x\neq 0. \tag{2.3}$$

The S-shaped demand function g capturing the trend following behavior is well documented in the HAM literature (see Chiarella et al., 2009), where the parameter β_c represents the extrapolation rate of the trend followers on the future price trend when the price deviation from the trend is small. In the following discussion, we let $g(x)=\tanh(\beta_c x)$, which satisfies the conditions in Eq. (2.3).

Among various price trends used in practice, weighted moving average rules are the most popular ones. In this paper, we assume that the price trend $u(t)$ of the trend followers at time t is measured by an exponentially decayed weighted average of historical prices over a time interval $[t-\tau, t]$ with time delay $\tau>0$, namely,

$$u(t) = \frac{1}{A}\int_{t-\tau}^{t} e^{-k(t-s)}P(s)ds,\ A = \frac{1-e^{-k\tau}}{k}, \tag{2.4}$$

where $k>0$ measures the decaying rate of the weights on the historical prices and A is a normalization constant. Note that the distributed delay used in Eq. (2.4) is not reducible for $0<\tau<\infty$. Eq. (2.4) implies that, when forming the price trend, the trend followers believe the more recent prices contain more information about the future price movements so

[1] To simplify the analysis, we assume that the market fractions are constant parameters as in the market fraction model in He and Li (2008), rather than dependent variables based on some performance measure, as in Brock and Hommes (1998). An extension along this line, as in Brock and Hommes (1998) or Dieci et al. (2006) in general, to allow investors switching between the two strategies is of interest and we leave this as future research.