Interval observers for LPV systems and application to the guaranteed state estimation of an induction machine

Stefan Krebs * Thomas Gellrich ** Sören Hohmann ***

* Institute of Control Systems, Karlsruhe Institute of Technology, Karlsruhe, Germany (e-mail: stefan.krebs@kit.edu)
** FZI Research Center for Information Technology, Karlsruhe, Germany, (e-mail: gellrich@fzi.de)
*** Institute of Control Systems, Karlsruhe Institute of Technology, Karlsruhe, Germany (e-mail: soeren.hohmann@kit.edu)

Abstract: The scope of this paper is the design of interval observers for linear parameter-varying systems subject to input and output uncertainties. To this, two coupled dynamical systems are designed which compute an upper bound and a lower bound of the state which enclose the real value with a bounded interval width. Besides of a full-order interval observer, a reduced-order interval observer is proposed to reduce the computational effort. In addition to sufficient conditions and the mathematical description of the interval observers, an optimization-based design procedure for their practical application is presented. The performance of the interval observers is shown by the guaranteed state estimation of an induction machine.

Keywords: Uncertain linear systems, Intervals, Reliability, State estimation, Induction machines

1. INTRODUCTION

Guaranteed state estimation methods which are based on the assumption that uncertainties are unknown but bounded while further parameters are not assumed to be known have mainly two advantages over other state estimation methods. Firstly, suitable error models can usually be found easily because boundaries of uncertain quantities are given directly in datasheets which is not the case when for example Kalman filters should be applied. In this case, the probability distributions of the uncertain quantities are rather used as tuning parameters than as a given quantification of the uncertainties. The second advantage of guaranteed state estimation methods is that they provide a guaranteed information on the estimated states which is of particular interest in the case of safety-critical systems (Heeks et al. (2002)). One example for such safety-critical systems are induction machines used as traction drives of electric vehicles. Interval observers are a class of guaranteed state estimation methods with a comparatively simple description of the uncertainties (namely interval vectors) which makes their real-time capable application possible. In contrast to interval observers, guaranteed state estimation methods with more complex descriptions of the uncertainties can provide a better inclusion but are sometimes too computational expensive for a real-time capable application (Zbranek and Vesely (2010)).

In this publication, the focus is on the design of interval observers for continuous-time linear parameter-varying (LPV) systems which has been treated before in a few publications (Chebotarev et al. (2015, 2013); Efimov et al. (2012); Thabet et al. (2013); Wang et al. (2015); Krebs et al. (2016)). The considered uncertainties are input and output uncertainties defined by bounded intervals while the parameters are assumed to be exactly known. The novelty of this publication which extends substantially the approach published in Krebs et al. (2016) is threefold. Firstly, the proof of stability is extended to coupled interval observers with a time-variant system matrix and secondly, a more general approach to compute the observer gain of the proposed interval observer is presented. The third contribution is the application of the full-order interval observer to the guaranteed state estimation of an induction machine. The full-order interval observer presented in aforementioned publication could not be designed for the induction machine due to too strict design conditions.

The paper is organized as follows: In section 2, lemmas are presented and operators are defined to facilitate the presentation of the main results. The problem statement in section 3 is followed by the results concerning the observer design in section 4. Afterwards, the results concerning the application of the presented approach to the guaranteed state estimation of an induction machine are presented in section 5, followed by a short conclusion in section 6.

2. PRELIMINARIES

In the following, multidimensional variables are represented by bold letters in order to avoid ambiguity because underlined variables denote lower bounds of intervals and overlined variables denote upper bounds of variables, i.e. $[J] = \underline{J}, \overline{J}$. Inequality relations between two multidimensional quantities, e.g. $\alpha \leq \beta$, are understood element-wise. The symbol $\preceq$ is used to indicate positive or negative
Lemma 1. (See Angeli and Sontag (2003)) A system

\[ \dot{x}(t) = A(t) \cdot x(t) + B(t) \cdot u(t) \]  

(1)

is called positive if \( A(t) \) is metzer and \( B(t) \cdot u(t) \) is nonnegative. For positive systems, \( \mathbb{R}_+^n \) is a positive invariant set with respect to (1).

Lemma 2. (See Rugh (1996)) System (1) with

\[ y(t) = C(t) \cdot x(t) \]  

(2)

is BIBO stable if its zero-input response is uniformly exponentially stable and there exist two finite constants \( \gamma \) and \( \xi \) such that

\[ \|B(t)\| \leq \gamma \]  

(3a)

\[ \|C(t)\| \leq \xi \]  

(3b)

holds.

Lemma 3. (See Rugh (1996)) The zero-input response of system (1) is uniformly exponentially stable if there exist a symmetric and continuously differential matrix function \( Q(t) \) and three positive constants \( \eta, \zeta, \nu \) such that

\[ \eta \cdot I \leq Q(t) \leq \zeta \cdot I \]  

(4a)

\[ A^T(t) \cdot Q(t) + Q(t) \cdot A(t) + \dot{Q}(t) \leq -\nu \cdot I \]  

(4b)

holds.

To be able to present the main results compactly, we introduce some matrix operations in Definition 1.

Definition 1.

\[ \widetilde{W} = (\widetilde{w}_{ij}) = M \{ W = (w_{ij}) \} \]  

(5)

\[ \widetilde{w}_{ij} = \begin{cases} w_{ij}, & \text{if } (i = j) \\ \{w_{ij}\}, & \text{otherwise} \end{cases} \]  

(6)

\[ \widetilde{P}^+ = (\widetilde{p}^+_{ij}) = \widetilde{P}^+ \{P = (p_{ij})\} \]  

(7)

\[ \widetilde{p}^+_{ij} = \begin{cases} p_{ij}, & \text{if } (i = j \vee (i \neq j \wedge p_{ij} \geq 0)) \\ 0, & \text{otherwise} \end{cases} \]  

(8)

\[ \widetilde{P}^- = (\widetilde{p}^-_{ij}) = \widetilde{P}^- \{P = (p_{ij})\} \]  

(9)

\[ \widetilde{p}^-_{ij} = \begin{cases} p_{ij}, & \text{if } (i \neq j \wedge p_{ij} < 0) \\ 0, & \text{otherwise} \end{cases} \]  

(10)

\[ M^+ = \max(0, M) = P^+ \{M\} \]  

(11)

\[ M^- = \min(0, M) = P^- \{M\} = M - M^+ \]  

(12)

3. PROBLEM STATEMENT

Consider the linear parameter-varying system

\[ \begin{cases} \dot{x}(t) = A(\theta_A(t)) \cdot x(t) + B(\theta_B(t)) \cdot u(t), \\ y(t) = C(\theta_C(t)) \cdot x(t), \\ x(0) = x_0. \end{cases} \]  

(13)

with \( A \in \mathbb{R}^{n \times n} \) being the system matrix depending on the time-varying parameters \( \theta_A(t) \), \( B \in \mathbb{R}^{n \times p} \) being the input matrix depending on the time-varying parameters \( \theta_B(t) \) and \( C \in \mathbb{R}^{q \times n} \) being the output matrix depending on the time-varying parameters \( \theta_C(t) \). \( u(t) \in \mathbb{R}^p \) denotes the input and \( x(t) \in \mathbb{R}^n \) is the state vector that can be divided into the measurable states \( y(t) \in \mathbb{R}^q \) and the unmeasurable states \( r(t) \in \mathbb{R}^{n-q} \):

\[ x(t) = \begin{bmatrix} y(t) \\ r(t) \end{bmatrix}. \]  

(14)

The values of \( \theta_A(t), \theta_B(t) \) and \( \theta_C(t) \) are assumed to be known for every \( t \), for example through measurements.

The aim of the guaranteed state estimation method to be presented is to determine a bounded interval \([\underline{r}(t), \overline{r}(t)]\) which includes \( r(t) \) in a guaranteed way, i.e.

\[ \underline{r}(t) \leq r(t) \leq \overline{r}(t), \quad \forall t \geq 0, \]  

(15a)

\[ r(t) - \underline{r}(t) < \epsilon, \quad \forall t \geq 0 \wedge \epsilon \in \mathbb{R}^n \]  

(15b)

under consideration of bounded parameter variances

\[ \theta_A(t) \in [\theta_{\underline{A}}, \theta_{\overline{A}}], \]  

(16a)

\[ \theta_B(t) \in [\theta_{\underline{B}}, \theta_{\overline{B}}], \]  

(16b)

\[ \theta_C(t) \in [\theta_{\underline{C}}, \theta_{\overline{C}}]. \]  

(16c)

4. MAIN RESULTS

After stating two assumptions, a full-order interval observer and a reduced-order interval observer will be presented to guarantee (15).

For the sake of notational simplicity, the parameter-dependency of the matrices is not displayed explicitly subsequently.

Assumption 1. There exist a symmetric, continuously differentiable and positive definite matrix function \( Q_{fo}(t) \in \mathbb{R}^{n \times n} \) and a matrix \( L_{fo}(t) \in \mathbb{R}^{n \times q} \) such that

\[ \Gamma_{fo}(t) = \frac{F_{fo}^T(t)}{F_{fo}(t)} \cdot Q_{fo}(t) + Q_{fo}(t) \cdot \frac{F_{fo}(t)}{F_{fo}(t)} + \dot{Q}_{fo}(t) \]  

(17)

with

\[ F_{fo}(t) = M \{ F_{fo}(t) \} \]  

(18)

under consideration of

\[ F_{fo}(t) = A(t) - L_{fo}(t) \cdot C(t) \]  

(19)

is negative definite.

Assumption 2. There exist a symmetric, continuously differentiable and positive definite matrix function \( Q_{ro}(t) \in \mathbb{R}^{(n-q) \times (n-q)} \) and a matrix \( L_{ro}(t) \in \mathbb{R}^{(n-q) \times q} \) such that

\[ \Gamma_{ro}(t) = \frac{F_{ro}^T(t)}{F_{ro}(t)} \cdot Q_{ro}(t) + Q_{ro}(t) \cdot \frac{F_{ro}(t)}{F_{ro}(t)} + \dot{Q}_{ro}(t) \]  

(20)

with

\[ F_{ro}(t) = M \{ F_{ro}(t) \} \]  

(21)

under consideration of

\[ F_{ro}(t) = A_{22}(t) - L_{ro}(t) \cdot A_{12}(t) \]  

(22)

is negative definite.

The matrices \( A_{12}(t) \in \mathbb{R}^{q \times (n-q)} \) as well as \( A_{22}(t) \in \mathbb{R}^{(n-q) \times (n-q)} \) are given by the block matrix

\[ A(t) = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{bmatrix}. \]  

(23)

Assumption 3. The elements of \( A(t) = (a_{ij}) \), the elements of \( B(t) = (b_{oi}) \) and the elements of \( C(t) = (c_{vik}) \) are bounded. i.e.

\[ |a_{ij}| < \infty, \quad \forall t \in \{1, \ldots, n\} \wedge \kappa \in \{1, \ldots, n\}, \]  

(24a)

\[ |b_{oi}| < \infty, \quad \forall o \in \{1, \ldots, n\} \wedge \kappa \in \{1, \ldots, p\}, \]  

(24b)

\[ |c_{vik}| < \infty, \quad \forall v \in \{1, \ldots, q\} \wedge \chi \in \{1, \ldots, n\}. \]  

(24c)

Remark 1. The assumptions 1 and 2 are adapted versions of the classical stability criterion given in lemma 3.
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