Bubbles and crashes: Gradient dynamics in financial markets

Daniel Friedman\textsuperscript{a,}\textsuperscript{*}, Ralph Abraham\textsuperscript{b}

\textsuperscript{a} Economics Department, University of California, Santa Cruz, CA 95064, USA
\textsuperscript{b} Mathematics Department, University of California, Santa Cruz, CA 95064, USA

\section*{Abstract}
Fund managers respond to the payoff gradient by continuously adjusting leverage in our analytic and simulation models. The base model has a stable equilibrium with classic properties. However, bubbles and crashes occur in extended models incorporating an endogenous market risk premium based on investors’ historical losses and constant-gain learning. When losses have been small for a long time, asset prices inflate as fund managers increase leverage. Then slight losses can trigger a crash, as a widening risk premium accelerates deleveraging and asset price declines.

\section*{1. Introduction}
Since their origin, financial markets have suffered from sporadic bubbles and crashes—episodes in which asset prices rise dramatically for no obvious reason, and later plummet (e.g., Penso de la Vega, 1688/1996; Mackay, 1841/1996). Recent examples include Japan’s stock and land price bubbles in the late 1980s, and the US dot.com and telecom bubbles in 2000. Such episodes are important as well as dramatic. As shorelines and river valleys are shaped largely by “100 year events,” so are financial markets, and the economy more generally. For example, the US Securities and Exchange Commission, the segregation of commercial banking from investment banking, and active monetary policy all arose in reaction to the 1929 US stock market crash and subsequent Great Depression (e.g., Kindleberger, 1978/1989/2000).

Despite their intrinsic interest, financial bubbles and crashes as yet have no widely accepted theoretical explanation. One reason is simply that they are sporadic. They seldom recur in the same country or market sector within the same generation of participants, so the data are problematic. A second reason is that established theoretical models maintain the assumption of financial market equilibrium. That assumption is difficult to reconcile with the dramatic episodes.

The present paper introduces new models and techniques for studying bubbles and crashes. The focus is on professional fund managers whose payoffs are the risk-adjusted returns they earn on their portfolios. Payoff maximization is not well defined outside equilibrium, so we assume that the managers continuously adjust their risk exposure so as to move up the current payoff gradient. Another non-standard feature is constant-gain learning (e.g., Cho et al., 2002), also known as exponential average expectations. An endogenous cost of risk is obtained from applying such learning to realized losses.

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\textsuperscript{*} Corresponding author. Tel.: +1831 459 4981; fax: +1831 459 5700.
E-mail addresses: dan@ucsc.edu (D. Friedman), abraham@vismath.org (R. Abraham).
Where possible, our models adapt and streamline standard ingredients. For example, we assume a single source of systematic risk, ignore inflation and taxes, and blur the distinctions between cash flow, earnings and dividends.

Section 2 spotlights some relevant stands of literature. Our models draw inspiration from one of the older strands, due to Keynes, Minsky and Kindleberger (KMK), and also from newer stands, especially the emerging agent-based approach.

Section 3 presents the basic model, beginning with the static ingredients and gradient dynamics. We characterize analytically a unique symmetric equilibrium and its comparative statics in the simplest version of the model. An agent-based simulation model illustrates and extends these results. The simulations converge reliably and smoothly to the equilibrium over a very wide range of parameter configurations.

Bubbles and crashes first appear in Section 4. It presents an extension incorporating some KMK-inspired features such as mean-reverting, manager-specific luck (or perhaps skill); investors who are constant-gain learners; and an endogenous risk cost. Analytic approximations suggest typical behavior and lead to conjectures about bubbles and crashes. Simulations confirm such episodes for a wide range of parameter configurations. The intuition is that when losses have been small for a long time, asset prices inflate as fund managers adopt riskier portfolios. When losses occur, as they eventually must, the more exposed managers get hit harder and sell faster. This puts downward pressure on asset prices, and an increasing risk premium accelerates the decline in asset price. This vicious cycle can drive asset price below fundamental value.

Bubbles and crashes are more prevalent in simulations when manager-specific luck is more volatile and longer-lived, when investors have shorter memories, when the economy grows faster and the discount rate is lower, and when current asset prices are higher.

Section 5 summarizes the results and suggests avenues for future research. Appendix A collects mathematical proofs, Appendix B describes extensions of the model, and Appendix C presents the KMK perspective. Additional material, including source and executable code for the simulations, can be found at http://www.vismath.org/research/landscapedyn/models/markets.

2. Existing literature

Modern financial economists define the fundamental value $V$ of an asset as the expected present value, given all available information, of the net cash flow the asset generates. The accepted definition of a bubble is a deviation of market price $P$ from $V$. Crashes are episodes when $B = P - V$ rapidly decreases from a positive value to a zero (or negative) value.

Beyond these simple definitions, consensus is elusive. Most early accounts of bubbles and crashes, e.g., Penso de la Vega and Mackay, emphasize the accompanying bursts of optimism and pessimism, and often seem to assign a causal role to “market psychology.” Absent some insight into (or preferably predictions of) how the bursts of optimism and pessimism arise, this approach does not seem very fruitful.

Some economists deny that bubbles exist, and assert that financial markets are always in equilibrium in the sense that $B = P - V = 0$. They explain famous historical episodes, such as Tulipmania in 17th century Netherlands, as just unusual moves in the fundamental value (e.g., Garber, 1989). Since $V$ is not directly observable, and because the episodes are so sporadic, it is hard to prove (or disprove) this view.

The “rational bubble” models of the 1980s proposed a rather different view (e.g., Blanchard and Watson, 1982; Tirole, 1982). The models allow no intertemporal arbitrage opportunity from one period to the next and traders have the same beliefs, but with an infinite horizon there might be a gap between $P$ and $V$ that grows at an exponential rate. A diverse collection of later papers ascribe bubbles to problems with information aggregation (e.g., Friedman and Aoki, 1992) or to interactions of rational traders with irrational traders (e.g., DeLong et al., 1990; Youssefmir et al., 1998; Brock and Hommes, 1998).

LeRoy (2004) concludes his integrative survey as follows.

We have considered four categories of accounts . . . [for recent apparent bubble and crash episodes]. As explanations, all four categories have problems. . . . Within the neoclassical paradigm there is no obvious way to derail the chain of reasoning that excludes bubbles. An alternative to the full neoclassical paradigm is to think about bubbles in a rational-agent setting—in particular to define fundamentals using the present-value relation—but to break off the analysis arbitrarily at some point rather than following the reasoning to implausible conclusions. The problems with this alternative are obvious: How does one write down formal models in such a setting? Where does one break off the analysis? Which conclusions from neoclassical analysis are to be accepted? We have no answers to these questions. . . . (p. 801)

The present paper resolves LeRoy’s conundrum by modelling financial markets that are not always in equilibrium. The agents always seek profit, and most of the time the market is near a steady state, but investors’ ongoing learning processes occasionally push the market far from equilibrium. Our modeling choices are guided partly by the empirical literature, and partly by the perspective of Minsky (1975) and Minsky (1982), who drew on ideas from Keynes (1936), later elaborated in Kindleberger (1978/1989/2000). Thus bubbles in our model are touched off by unusual runs of luck by some managers. As the bubble inflates, losses are rare and dazzled investors allow risk premiums to shrink, leading to still higher asset prices. When losses finally do occur, risk premiums expand and asset prices decline, producing wider losses, wider risk premiums, and further losses. Appendix C spells out the KMK perspective on bubbles and crashes.
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