A generalized smith predictor for unstable time-delay SISO systems

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1. Introduction

Time delays appear naturally in control applications. They can be either intrinsic to the physical process to be controlled or originated in the implementation of the feedback loop [1]. Furthermore, industrial processes usually operate in a fixed set-point during long periods of time and thus disturbance rejection is a fundamental issue.

An LTI time-delay SISO process subject to input disturbances can be described by

\[ y(s) = G(s)e^{-hs}[u(s) + w(s)] = \hat{y}(s)e^{-hs} \tag{1} \]

where \( y \in \mathbb{R} \) is the measurable output, \( \hat{y} \in \mathbb{R} \) is the unmeasurable non-delayed output, \( u \in \mathbb{R} \) is the control input, \( w \in \mathbb{R} \) is an input disturbance, \( h \geq 0 \) is a constant time delay and \( G(s) = C(sl - A)^{-1}B \) is referred to as the delay-free system.

When controlling a time-delay system, an ideal scenario is depicted in Fig. 1. It is “ideal” in the sense that the delay is pushed out of the feedback loop, the non-delayed output \( \hat{y} \) is available, and thus the controller \( K(s) \) can be simply designed for the rational part of the model, \( G(s) \), using conventional techniques. Since \( \hat{y} \) is not accessible, a reasonable approach consists of constructing an output prediction \( \hat{y} \), so that it can be used to control the system as in the ideal scenario. The prediction should be based on the available input/output information, having the following structure:

\[ \hat{y}(s) \triangleq F_1(s)u(s) + F_2(s)y(s) \tag{2} \]

where the filters \( F_1(s) \) and \( F_2(s) \) must be stable and derived from the model.

In the seminal work [2], the Smith Predictor (SP) makes use of the filter \( F_2^S(s) \triangleq G(s) - G(s)e^{-hs} \), sometimes referred to as the SP block, whereas \( F_1^S(s) \triangleq 1 \). It is easy to verify that the prediction \( \hat{y}_p(s) \triangleq F_1^S(s)u(s) + F_2^S(s)y(s) \) satisfies \( \hat{y}_p(s) = G(s)u(s) = y(s) \), if there are no disturbances. Indeed, the SP removes the delay element from the denominators of all the closed-loop sensitivity functions, reducing the control problem to that of a delay-free system. The methodology described above has been referred to as the “Smith’s Principle” in the literature. However, the SP cannot be applied to open-loop unstable plants and regardless of the main controller, only constant disturbances can be rejected [3].

Many structures, commonly referred to as dead-time compensators (DTCs), have been developed to mitigate these issues [4,5], either to achieve load disturbance rejection for pure integrating processes with long dead-time [6–12], or to control unstable time-delay systems [13–21]. Some works have been also focused on counteracting periodic disturbances [22–24]. These schemes commonly have an inner stabilizing loop and employ more controllers. Furthermore, most solutions are highly specific on the control goals and/or the plant structure, and they fail in completely removing the delay element from the feedback loop, making the design process more complicated. To the best of the author’s knowledge, for integrating and unstable systems, none of the aforementioned works except those proposed in [6,18,19], fulfill the Smith’s Principle. Next, these schemes are reviewed and recast under the same structure, in order to place the present work in context.
1.1. The Smith’s principle

As aforementioned, few schemes have been proposed to generalize the Smith Predictor to unstable systems, avoiding the instability of the predictor block while fulfilling the Smith’s Principle. The first attempt in this direction can be found in [6]. In that work, the SP block was modified by choosing \( F_{\text{MSP}}(s) \equiv G(s) - G(s)e^{-ah} \) and \( F_{\text{MSP}}(s) = 1 \), where \( G(s) \equiv Ce^{-ah}sl - A_1^1B \). However, it was later where this approach was generalized and named as the Modified Smith Predictor (MSP) [25]. The key feature of this scheme is that the MSP block can be computed in the time domain as \( \mathcal{L}^{-1}[F_{\text{MSP}}(s)u(s)] = Ce^{-ah} \int_0^b e^{At}Bu(t - \xi) d\xi \), which is a definite integral and therefore, stable. Regarding disturbance rejection, the MSP alters the low frequency gain of the primary controller because it has non-zero static gain, that is, \( F_{\text{MSP}}(0) \neq 0 \). Consequently, constant disturbances cannot be rejected even if the primary controller contains integral action. This drawback was already addressed in [6] by choosing \( G(s) = -C_0^h e^{-Ah} d\xi + Ce^{-ah}sl - A_1^1B \), with the inconvenient that \( G(s) \) is no longer strictly-proper and the corresponding controller may be more complicated.

Other proposals were developed inspired on the discrete-time framework. In [19], the SP was complemented with an additional filter, \( F_{\text{FSP}}(s) \equiv F_{\text{FSP}}(s) \), leading to the Filtered Smith Predictor (FSP). The resulting predictor block was \( F_{\text{FSP}}(s) \equiv G(s) - G(s)F_{\text{FSP}}(s)e^{-ah} \), where the new filter \( F_{\text{FSP}}(s) \) played a key role, being used to avoid the unstable modes in \( F_{\text{FSP}}(s) \). In continuous-time, this pole-zero cancellation cannot be performed by the use of polynomial division because the numerator of \( F_{\text{FSP}}(s) \) is a non-rational expression. However, in the discrete-time framework, this can be done analytically by solving a Diophantine equation. In the same process, the block can be also adjusted to reject any class of disturbances [26,27].

The Generalized Predictor (GP), originally proposed in [18], was originated from a discrete-time reasoning. However, the formulation next presented is developed in continuous-time for the sake of comparison. Similarly to the MSP, the instability of the GP block was avoided by selecting \( F_{\text{GP}}(s) \equiv G(s) - G(s)e^{-ah} \) with \( G(s) \equiv Ce^{-ah}sl - A_1^1B \), whereas \( F_{\text{GP}}(s) = G(s)G(s) \). As a result, the GP block can be computed as \( \mathcal{L}^{-1}[F_{\text{GP}}(s)u(s)] = C_0^h e^{At}Bu(t - \xi) d\xi \), which is a stable block. In order to cancel the effect of constant disturbances, the GP made use of an extra loop, making the analysis more complicated [28].

The schemes previously reviewed lead to a control structure as depicted in Fig. 2, with filters given in Table 1.

1.2. Contribution

In this paper, with special emphasis on transparency and design simplicity of the resulting control strategy, a generalization of the SP is proposed to solve the following problem:

![Fig. 1. An ideal control loop (unfeasible).](image1)

![Fig. 2. A general structure for predictor-based control schemes.](image2)

Table 1

<table>
<thead>
<tr>
<th>Scheme</th>
<th>( F_{\text{FSP}}(s) )</th>
<th>( F_{\text{GP}}(s) )</th>
<th>Proposed in</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>( G(s) - G(s)e^{-ah} )</td>
<td>1</td>
<td>[2]</td>
</tr>
<tr>
<td>MSP</td>
<td>( G(s) - G(s)e^{-ah} )</td>
<td>1</td>
<td>[6]</td>
</tr>
<tr>
<td>FSP</td>
<td>( G(s) - G(s)e^{-ah} )</td>
<td>( F_{\text{FSP}}(s) )</td>
<td>[19]</td>
</tr>
<tr>
<td>GP</td>
<td>( G(s) - G(s)e^{-ah} )</td>
<td>( G(s)G(s) )</td>
<td>[18]</td>
</tr>
</tbody>
</table>

Problem 1. Consider a controller \( K \) designed to meet some requirements based on the delay-free loop depicted in Fig. 1. Then, find a predictor, that is, design filters \( F_1 \) and \( F_2 \), such that the same controller \( K \) in Fig. 2:

A) guarantees internal stability
B) achieves the same nominal tracking performance
C) achieves rejection of the same type of disturbances

2. Problem reformulation

As already mentioned, a celebrated feature of the SP is that it exactly reduces the control problem to its delay-free counterpart, by constructing an “exact” prediction. In what follows, a prediction \( \hat{y}(s) \) for the system (1) is said to be exact if \( \hat{y}(s) = y(s) \) hold in the nominal case. It is easy to show that a prediction computed by (2) is exact if and only if

\[
F(s) = \left(1 - F_2(s)e^{-ah}\right)G(s).
\]

The main advantage of obtaining an exact prediction is that the design and analysis of the resulting control-loop are drastically simplified, which is a highly appreciated feature of the original SP. This is formally stated by the following proposition:

Proposition 1. If the output prediction computed by (2) is exact, then the input-output transfer functions of the predictor - based control loop depicted in Fig. 2 satisfy:

\[
G_{\text{r},y}(s) = \hat{G}_{\text{r},y}(s)
\]

\[
G_{\text{ru},y}(s) = \hat{G}_{\text{ru},y}(s) + \hat{G}_{\text{r},y}(s)F_{\text{F}}(s)
\]

\[
G_{\text{ru},n}(s) = \hat{G}_{\text{ru},n}(s)F_{\text{F}}(s)
\]

\[
G_{\text{r},n}(s) = \hat{G}_{\text{r},n}(s)F_{\text{F}}(s)
\]

where \( \hat{G}_{\text{r},y}, \hat{G}_{\text{ru},y}, \hat{G}_{\text{r},n}, \hat{G}_{\text{ru},n} \) are the input-output transfer functions of the ideal loop in Fig. 1.

1 Here \( \mathcal{L}^{-1} \) denotes the inverse Laplace transform operator.
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