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On entropy, financial markets and minority games

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ABSTRACT

The paper builds upon an earlier statistical analysis of financial time series with Shannon information entropy, published in [L. Molgedey, W. Ebeling, Local order, entropy and predictability of financial time series, European Physical Journal B—Condensed Matter and Complex Systems 15/4 (2000) 733–737]. A novel generic procedure is proposed for making multistep-ahead predictions of time series by building a statistical model of entropy. The approach is first demonstrated on the chaotic Mackey–Glass time series and later applied to Japanese Yen/US dollar intraday currency data. The paper also reinterprets Minority Games [E. Moro, The minority game: An introductory guide, Advances in Condensed Matter and Statistical Physics (2004)] within the context of physical entropy, and uses models derived from minority game theory as a tool for measuring the entropy of a model in response to time series. This entropy conditional upon a model is subsequently used in place of information-theoretic entropy in the proposed multistep prediction algorithm.

1. Introduction

In many areas of science and engineering a common recurring task arises: forecasting ΔT steps ahead in an observed time series $x_1, x_2, \dots, x_N$ of some physical phenomena. Established forecasting techniques typically fall into two categories: *time domain* analysis and *state–space* modelling. Usually the process may involve finding a linear or non-linear regression function $f(\cdot)$ that takes as arguments past values of x_t:

$$x_{t+\Delta T} = f\left(x_t, x_{t-1}, x_{t-2}, \dots, x_{t-p+1}\right) + \varepsilon \quad (1)$$

with ε accounting for noise and p denoting the number of predictor variables. However, in cases where observations $\{x_i\}$, also referred to as x_t with integer time steps t, come from financial markets for example foreign exchange currency fluctuations, the Eq. (1) fails to capture sufficiently the underlying time series generator and its often non-stationary nature. The art of predicting financial time series is notorious for being unreliable and difficult, which is evidenced by poor out-of-sample performance of models yielding apparently good in-sample fits [1]. In fact, simple Random Walk models make a good job of describing, but not forecasting values of financial time series, which is consistent with the Efficient Market Hypothesis. The inadequacy of the existing approaches has been mentioned before in for example [2,3], and some explanation of the perceived failures of the status quo has been contained in the works of Mandelbrot [4]. Perhaps in the absence of a compelling alternative the mainstream econometrics modelling framework (computational finance) assumes that prices of financial assets follow a simple stochastic Random Walk process [5]. However, financial time series are not generated by a Random

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Walk model or a linear autoregressive process. Instead they arise as a result of interactions between a large number of adaptive traders which provides a justification for applying various tools of statistical physics to computational finance [6]. Instead of order, sought after by mainstream econometricians through economic theories, a natural state for financial markets seems to be characterised by disorder, punctuated by brief periods of ordered behaviour [7]. Because of failures of conventional approaches some scientists, most notably from the statistical physics community, have turned to using entropy as a tool for analysing financial time series [3]. As another example, in Refs. [7,8] Molgedey and Ebeling consider the predictability of financial time series by extracting Shannon n-gram (block) entropies from the Dow Jones Industrial Average and the German stock exchange DAX futures data.

In the search for potentially viable alternatives to applying Eq. (1) directly, in subsequent sections this paper takes the entropy-based approach further by proposing a generic method for indirectly forecasting changes in time series ΔT steps ahead. The approach is first demonstrated using the chaotic Mackey–Glass time series, subsequently to be followed by a similar analysis of the Japanese Yen/US Dollar intraday currency futures data. Weaknesses of the method are highlighted and a possible improvement is suggested in the form of replacing the information theoretic entropy with a physical entropy extracted from minority game theory models [9].

2. Forecasting with entropy

We assume that there are N observations $x_1, x_2, \ldots, x_N$ available from some physical process. The goal of the analysis is to produce an estimate for the change $x_{t+\Delta T} - x_t$, where $t \in \mathbb{I}$ denotes the current time step and ΔT is the forecasting horizon. The proposed algorithm can be outlined in the following sequence of steps:

(1)	start with an observation sequence leading up to x_t
	$\Downarrow$
(2)	extract entropy time series $H(t)$ from x_t
	$\Downarrow$
(3)	assume a statistical model for entropy
	$\Downarrow$
(4)	simulate possible future paths of x_{t+k} ΔT steps ahead
	$\Downarrow$
(5)	measure entropy for the simulated paths
	$\Downarrow$
(6)	discard paths inconsistent with entropy model identified in (3)
	$\Downarrow$
(7)	formulate a forecast for $x_{t+\Delta T}$ based on the retained paths

With respect to estimating entropy $H(t)$ of a given time series, a good starting point might be the Shannon n-gram (block) entropy suggested in Refs. [7,8]. In a very general case, a given sequence of N observations $x_1, x_2, \ldots, x_N$ is first partitioned into subvectors of length L with an overlap of one time step, which are further divided into subtrajectories (delay vectors) of length $n < L$. Real-valued observations $x_i \in \mathbb{R}$ are discretised by mapping them onto λ non-overlapping intervals $A^{\{\lambda\}}(x_i)$. The precise choice of those intervals (also called *states*) denoted by $A^{\{\lambda\}}$ would depend on the range of values taken by x_i.[1] Hence a certain subtrajectory $x_1, x_2, \ldots, x_n$ of length n can be represented by a sequence of states $A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}$. The authors then define the n-gram entropy (entropy per block of length n) to be

$$H_n = -\sum_{\Omega} p\left(A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}\right) \log_\lambda p\left(A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}\right). \tag{2}$$

In the above equation the summation is done over all possible state sequences $\Omega \in \{A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}\}$. The probabilities $p\left(A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}\right)$ are calculated based on all subtrajectories $x_1, x_2, \ldots, x_n$ contained within a given subvector of the length L. Predictability of the time series, expressed as an uncertainty of predicting the next step given the past n states $A^{\{\lambda\}}$, is given by a conditional (dynamic) entropy (or differential block entropy)

$$h_n = H_{n+1} - H_n. \tag{3}$$

In the actual analysis [7,8] of financial time series, instead of using the dynamic entropy as per Eq. (3) the authors use a local value of the uncertainty defined [7] as

$$h_n^{(1)} = -\sum_{\Omega'} p\left(A_{n+1}^{\{\lambda\}} | A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}\right) \log_\lambda p\left(A_{n+1}^{\{\lambda\}} | A_1^{\{\lambda\}}, A_2^{\{\lambda\}}, \ldots, A_n^{\{\lambda\}}\right), \tag{4}$$

[1] In subsequent sections financial time series will be used. In case of modelling financial returns the simplest choice of states $A^{\{\lambda\}}$ can be made by setting $\lambda = 2$. Then $A^{\{up\}}$ corresponds to positive returns and $A^{\{down\}}$ would represent negative returns.