On entropy, financial markets and minority games

Christopher A. Zapart

The Graduate University for Advanced Studies, Department of Statistical Sciences, The Institute of Statistical Mathematics, 4-6-7 Minami-Azabu, Minato-ku, Tokyo 106-8569, Japan

ARTICLE INFO

Article history:
Received 10 June 2008
Received in revised form 17 November 2008
Available online 11 December 2008

PACS:
89.70.Cf
89.65.Gh
89.75.-k

Keywords:
Entropy
Econophysics
Minority games
High-frequency financial time series
Time series prediction
Foreign exchange currency markets

ABSTRACT


© 2008 Elsevier B.V. All rights reserved.

1. Introduction

In many areas of science and engineering a common recurring task arises: forecasting \( \Delta T \) steps ahead in an observed time series \( x_1, x_2, \ldots, x_N \) of some physical phenomena. Established forecasting techniques typically fall into two categories: time domain analysis and state–space modelling. Usually the process may involve finding a linear or non-linear regression function \( f(\cdot) \) that takes as arguments past values of \( x_t \):

\[
x_{t+\Delta T} = f \left( x_t, x_{t-1}, x_{t-2}, \ldots, x_{t-p+1} \right) + \varepsilon
\]

with \( \varepsilon \) accounting for noise and \( p \) denoting the number of predictor variables. However, in cases where observations \( \{x_i\} \), also referred to as \( x_t \) with integer time steps \( t \), come from financial markets for example foreign exchange currency fluctuations, the Eq. (1) fails to capture sufficiently the underlying time series generator and its often non-stationary nature. The art of predicting financial time series is notorious for being unreliable and difficult, which is evidenced by poor out-of-sample performance of models yielding apparently good in-sample fits [1]. In fact, simple Random Walk models make a good job of describing, but not forecasting values of financial time series, which is consistent with the Efficient Market Hypothesis. The inadequacy of the existing approaches has been mentioned before in for example [2,3], and some explanation of the perceived failures of the status quo has been contained in the works of Mandelbrot [4]. Perhaps in the absence of a compelling alternative the mainstream econometrics modelling framework (computational finance) assumes that prices of financial assets follow a simple stochastic Random Walk process [5]. However, financial time series are not generated by a Random
Walk model or a linear autoregressive process. Instead they arise as a result of interactions between a large number of adaptive traders which provides a justification for applying various tools of statistical physics to computational finance [6]. Instead of order, sought after by mainstream econometricians through economic theories, a natural state for financial markets seems to be characterised by disorder, punctuated by brief periods of ordered behaviour [7]. Because of failures of conventional approaches some scientists, most notably from the statistical physics community, have turned to using entropy as a tool for analysing financial time series [3]. As another example, in Refs. [7,8] Molgedey and Ebeling consider the predictability of financial time series by extracting Shannon n-gram (block) entropies from the Dow Jones Industrial Average and the German stock exchange DAX futures data.

In the search for potentially viable alternatives to applying Eq. (1) directly, in subsequent sections this paper takes the entropy-based approach further by proposing a generic method for indirectly forecasting changes in time series $\Delta T$ steps ahead. The approach is first demonstrated using the chaotic Mackey–Glass time series, subsequently to be followed by a similar analysis of the Japanese Yen/US Dollar intraday currency futures data. Weaknesses of the method are highlighted and a possible improvement is suggested in the form of replacing the information theoretic entropy with a physical entropy extracted from minority game theory models [9].

2. Forecasting with entropy

We assume that there are $N$ observations $x_1, x_2, \ldots, x_N$ available from some physical process. The goal of the analysis is to produce an estimate for the change $x_{t+\Delta T} - x_t$, where $t \in \mathbb{I}$ denotes the current time step and $\Delta T$ is the forecasting horizon. The proposed algorithm can be outlined in the following sequence of steps:

1. start with an observation sequence leading up to $x_t$
2. extract entropy time series $H(t)$ from $x_t$
3. assume a statistical model for entropy
4. simulate possible future paths of $x_{t+k} \Delta T$ steps ahead
5. measure entropy for the simulated paths
6. discard paths inconsistent with entropy model identified in (3)
7. formulate a forecast for $x_{t+\Delta T}$ based on the retained paths

With respect to estimating entropy $H(t)$ of a given time series, a good starting point might be the Shannon n-gram (block) entropy suggested in Refs. [7,8]. In a very general case, a given sequence of $N$ observations $x_1, x_2, \ldots, x_N$ is first partitioned into subvectors of length $L$ with an overlap of one time step, which are further divided into subtrajectories (delay vectors) of length $n < L$. Real-valued observations $x_t \in \mathbb{R}$ are discretised by mapping them onto $\lambda$ non-overlapping intervals $A^{(k)}(x_t)$. The precise choice of those intervals (also called states) denoted by $A^{(k)}$ would depend on the range of values taken by $x_t$. Hence a certain subtrajectory $x_1, x_2, \ldots, x_n$ of length $n$ can be represented by a sequence of states $A^{(1)}_1, A^{(2)}_2, \ldots, A^{(n)}_n$. The authors then define the $n$-gram entropy (entropy per block of length $n$) to be

$$H_n = -\sum_{\Omega} p(A^{(k)}_1, A^{(k)}_2, \ldots, A^{(k)}_n) \log p(A^{(k)}_1, A^{(k)}_2, \ldots, A^{(k)}_n).$$

In the above equation the summation is done over all possible state sequences $\Omega \in \{A^{(k)}_1, A^{(k)}_2, \ldots, A^{(k)}_n\}$. The probabilities $p(A^{(k)}_1, A^{(k)}_2, \ldots, A^{(k)}_n)$ are calculated based on all subtrajectories $x_1, x_2, \ldots, x_n$ contained within a given subvector of the length $L$. Predictability of the time series, expressed as an uncertainty of predicting the next step given the past $n$ states $A^{(k)}$, is given by a conditional (dynamic) entropy (or differential block entropy)

$$h_n = H_{n+1} - H_n.$$  

In the actual analysis [7,8] of financial time series, instead of using the dynamic entropy as per Eq. (3) the authors use a local value of the uncertainty defined [7] as

$$h^{(1)}_n = -\sum_{\Omega'} p(A^{(k)}_{n+1}|A^{(k)}_1, A^{(k)}_2, \ldots, A^{(k)}_n) \log p(A^{(k)}_{n+1}|A^{(k)}_1, A^{(k)}_2, \ldots, A^{(k)}_n),$$

In subsequent sections financial time series will be used. In case of modelling financial returns the simplest choice of states $A^{(k)}$ can be made by setting $\lambda = 2$. Then $A^{(up)}$ corresponds to positive returns and $A^{(down)}$ would represent negative returns.

In Refs. [7,8] Molgedey and Ebeling consider the predictability of financial time series by extracting Shannon n-gram (block) entropies from the Dow Jones Industrial Average and the German stock exchange DAX futures data.
دریافت فوری متن کامل مقاله

امکان دانلود نسخه تمام متن مقالات انگلیسی
امکان دانلود نسخه ترجمه شده مقالات
پذیرش سفارش ترجمه تخصصی
امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
امکان دانلود رایگان ۲ صفحه اول هر مقاله
امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
دانلود فوری مقاله پس از پرداخت آنلاین
پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات