Risk-coherent $\mathcal{H}_2$-optimal disturbance rejection under model uncertainty

Matias I. Müller * Patricio E. Valenzuela * Cristian R. Rojas *

* Department of Automatic Control, School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden (e-mail: \{mimr2, pva, crrro\}@kth.se)

Abstract: A control design procedure for disturbance rejection, when the disturbance model is uncertain, is proposed. We use the probabilistic information about the process disturbance model to design a controller to account for the uncertainty by using a risk-theoretical approach. By introducing the notion of coherent measures of risk, we analyze standard approaches to account for this uncertainty, and we intend to show that the conditional value-at-risk (CVaR) is an appropriate function to measure the uncertainty in the disturbance model. We also derive a convex formulation for the controller design problem when the Youla parameter is linearly parametrized. A numerical example illustrates the main discussion of this article.

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1. INTRODUCTION

From a classical perspective, control systems are designed so that the closed loop attains a desired performance. Some of the many traditional frameworks used for control design are: the linear quadratic regulator (LQR) (Kwakernaak and Sivan, 1972), linear quadratic Gaussian (LQG) control (Söderström, 2002) and model predictive control (Rawlings and Mayne, 2009). One of the most relevant indices to measure the loop performance is the $\ell_2$-norm of the tracking error signal (Doyle et al., 2009). The controller design is based on the previous information we have about the system we want to control. Such information can be obtained from different methods such as physical modelling, experiments or even previous knowledge, where uncertainty may be introduced.

Uncertainty can be seen as the lack of knowledge to fully describe a phenomenon, and causes several problems when trying to take the best decision under limited information. In the control framework, the task of designing a control law might be difficult due to different sources of uncertainty. We can distinguish two of them: uncertainty in the transfer functions composing the system (which we call modeling error) and uncertainty in the external signals affecting the system, namely disturbances and noise (which we group under the name process disturbance). Both sources of uncertainty are present in the different steps of the control design.

The process disturbance can be interpreted in different ways, e.g., physical disturbances and/or measurement noise. The design under the presence of process disturbances can be addressed as a regulation problem (Saberi et al., 2000) and it is usually solved, in an $\ell_2$ framework, by imposing an extra penalty on the performance cost related to the frequency-domain properties of the process disturbance. These properties can generate a trade-off between tracking performance and disturbance rejection in the design (Chen, 2003).

On the other hand, the modeling error is related to how accurate the information we have about the true process (usually condensed in models) is. This subject has been widely studied in the field of robust control. In this line Zhou et al. (1996) and Bhattacharyya et al. (1995) explain how to compensate for our lack of knowledge in terms of fundamental limitations over tracking performance and stability robustness. In robust control, the modeling error problem is compensated by analyzing the worst-case scenarios, where a formal framework, such as $\mathcal{H}_\infty$–control (Francis, 1987), can be used. In this article, we will deal with the error in the model for the process disturbance.

The problem of properly measuring the uncertainty has been studied in the theory of risk in finance, where in Markowitz (1959) this notion was originally applied to portfolio optimization. To appropriately measure the risk associated with financial portfolios, the notion of coherent measures of risk has been introduced by Artzner et al. (1999). This family of measures has many attractive properties, such as convexity and the fact that it encourages diversification. The latter means that, in a portfolio setting, it is always better to invest in several assets rather than in a single one, which is a well known and intuitive strategy to reduce risk in portfolios. Along this work, we discuss how the notion of coherent measures of risk can be employed in control design. Some coherent measures of risk are the expectation and the worst-case scenario. A specific measure of risk, known as conditional value-at-risk (CVaR) (Rockafellar and Uryasev, 2000), has been...
recently employed by Van Parys et al. (2016) as convex constraints in linear-quadratic control problems.

We remark that the coherent measures of risk approach is one of many existing alternatives to measure uncertainty. Indeed, the notion of risk is not new in control theory and several frameworks have been developed. One of the most important ones is the risk-sensitive criterion introduced by Jacobson (1973) and polished by Whittle (1990). Works as Liu et al. (2003) address the problem of minimizing the risk according to the expected value of the weighted tracking error performance, while Ito et al. (2016) use this notion to study the infinite-horizon control problem.

The main contribution of this work is to propose an alternative framework for control design when there is uncertainty in the parameters modeling the process disturbance. We discuss how the notion of coherent measures of risk can be employed in this setting, and we intend to obtain a systematic approach to account for the uncertainty in the control design problem. The goal is to obtain a controller that minimizes the risk of falling into low closed-loop tracking performance, by minimizing its conditional value-at-risk, and to compare it with traditional designs such as average risk, worst-case scenario, and the nominal design.

The paper is structured as follows. Section 2 defines the problem in the context of this article. Section 3 formalizes what we mean by risk and how to measure it. Section 4 describes an algorithm to synthesize the controller by solving a quadratically-constrained linear programming (QCLP) problem. Section 5 shows a numerical example comparing the closed-loop performance under controllers derived from different risk measures. Finally, conclusions are presented in Section 6.

1.1 Notation

The sets of natural, real and complex numbers are \( \mathbb{N} \), \( \mathbb{R} \) and \( \mathbb{C} \), respectively. The one-step time shift operator, in discrete-time, is denoted by \( q \). We will use the vectors \( 1_N := [1 \ 1 \ \cdots \ 1]^T \in \mathbb{R}^N \) and \( 0_N := 0 \cdot 1_N \in \mathbb{R}^N \). Any function \( F(q, \theta) \) represents a real rational proper transfer function in \( q \) parametrized by \( \theta \in \mathbb{R}^m \). \( E\{\cdot\} \) stands for the expected value. \( X^+ := \max\{0, X\} \). A sequence of random variables \( \{y_n\} \) that converges almost surely towards the random variable \( y \) is denoted as \( y_n \xrightarrow{a.s.} y \). \( x \sim p(x) \) denotes that \( x \) is distributed according to the probability density function (pdf) \( p(x) \), while \( \text{supp}(p) \) stands for the support of \( p(x) \), i.e., the closure of the set \( \{x : p(x) \neq 0\} \). \( \mathbb{L}^2 \) is the Hilbert space composed of all random variables with bounded mean and variance.

The set of all real rational stable proper transfer functions is \( \mathcal{RH}_\infty \). \( \mathcal{RL}_2 \) is defined as the space of real rational functions \( F : \mathbb{C} \to \mathbb{C} \), analytic on \( \Pi := \{z \in \mathbb{C} : |z| = 1\} \), such that \( \|F\|_2 < \infty \), and inner product defined as

\[
\langle G, F \rangle := \frac{1}{2\pi} \int_{\Pi} G(z) F(z^{-1}) z^{-1} \, dz,
\]

and

\[
\|F\|_2^2 := \frac{1}{2\pi} \int_{\Pi} F(z) F(z^{-1}) z^{-1} \, dz,
\]

where \((\cdot)^H \) denotes the conjugate transpose operator. The transpose is denoted as \((\cdot)^T \). For a discrete-time second-order asymptotically wide-sense stationary (wss) random signal \( \{x(t)\} \), its norm is defined as

\[
\|x\|_2^2 := \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} E \{x^2(t)\}.
\]

2. PROBLEM DEFINITION

Consider two discrete-time linear and time-invariant (LTI) filters \( G_0(q) \) and \( H_0(q) \), as depicted in Fig 1, describing the system \( y(t) = G_0(q)u(t) + H_0(q)n(t) \), where \( \{n(t)\} \) is a zero-mean white signal with variance \( \sigma_n^2 \). Under this setting, we assume the following:

**Assumption 1.** The system \( G_0(q) \) is known with sufficient accuracy for control design purposes. We capture the uncertainty of \( H_0(q) \) in a random variable \( \theta \in \mathbb{L}^2 \) with pdf \( p(\theta) \) of bounded support \( \Theta \), such that \( H_0(q) = H(q, \theta) \) for a known structure \( H(q, \theta) \). It is assumed that \( G_0(q) \) is stable and strictly proper, while \( \theta \) is such that \( H(q, \theta) \) is a biproper minimum-phase stable filter with probability 1.

Assumption 1 requires that the information about the plant is accurate enough for the controller design, i.e., there is no uncertainty in \( G_0(q) \). Also, since \( G_0(q) \) is already stable, the stabilization problem is not an issue here.

**Remark 1.** The reasoning behind why \( \theta \) is random can be understood within a Bayesian framework. The pdf \( p(\theta) \) denotes our knowledge about \( \theta \), which comes from prior knowledge and possibly experimental data collected from the true system. The problem of obtaining \( p(\theta) \) might be difficult, and it is beyond the scope of this work. However, it can be addressed by using approximate Bayesian techniques such as Markov Chain Monte Carlo (MCMC) methods (Robert and Casella, 2004).

The implementation of the controller is as depicted in Fig. 1, where the reference signal \( \{r(t)\} \) is zero-mean and wss with spectral factor \( \Omega_r(z) \) (Francis, 1987, Chapter 7), such that \( r \) and \( n \) are statistically independent. Hence, the norm of the tracking error \( e \) can be written as

\[
\|e\|_2^2 = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} E \{e^2(t) | \theta\}
\]

where \( \frac{G_0 \Omega_r}{1+G_0 \Omega_r} - 1 \) represents the biproper minimum-phase stable filter with probability 1.
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