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The stochastic bifurcation behaviour of speculative financial markets[☆]

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Abstract

This paper establishes a continuous-time stochastic asset pricing model in a speculative financial market with fundamentalists and chartists by introducing a noisy fundamental price. By application of stochastic bifurcation theory, the limiting market equilibrium distribution is examined numerically. It is shown that speculative behaviour of chartists can cause the market price to display different forms of equilibrium distributions. In particular, when chartists are less active, there is a unique equilibrium distribution which is stable. However, when the chartists become more active, a new equilibrium distribution will be generated and become stable. The corresponding stationary density will change from a single peak to a crater-like density. The change of stationary distribution is characterized by a bimodal logarithm price distribution and fat tails. The paper demonstrates that stochastic bifurcation theory is a useful tool in providing insight into various types of financial market behaviour in a stochastic environment.

Keywords: Heterogeneous agents; Speculative behaviour; Random dynamical systems; Stochastic bifurcations; Invariant measures

1. Introduction

The theory of random dynamical systems provides a very powerful mathematical tool for understanding the limiting behaviour of stochastic systems. Recently, it has been applied to economics and finance to help in understanding the stochastic nature of financial models with random perturbations. In particular, the study of the limiting distribution of various stochastic models in economics and finance gives a good description of stationary markets. For example, Föllmer et al. [9] study the existence and uniqueness of the limiting distribution in a discrete financial market model with different expectations through stochastic learning and Böhm and Chiarella [4] investigate the long-run behaviour (stationary solutions) for mean-variance preferences under various predictors. Those models mainly focus on the existence, uniqueness and stability of limiting distributions of discrete-time financial models, rather than the changes

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of existence and stability of multiple limiting distributions of continuous-time financial models as their parameters change.

Stochastic bifurcation theory has been developed to study the changes of existence and stability of limiting distributions. There seems to have been no application of it to heterogeneous agent models of continuous-time financial markets except the one-dimensional continuously randomized version of Zeeman's [15] stock market model studied by Rheinlaender and Steinkamp [12]. These authors show a stochastic stabilization effect and possible sudden trend reversal in the financial market. For higher dimensional financial market models with heterogeneous agents in the continuous-time framework, the application of stochastic bifurcation theory faces many challenges. In this paper, we take the very basic heterogeneous agent financial market model of fundamentalists and chartists developed by Beja and Goldman [3] and Chiarella [5] and set it up as a two-dimensional stochastic model by introducing a noisy fundamental price in a continuous-time framework. We then use stochastic bifurcation theory to analyze numerically the changes and stability of multiple limiting distributions of the two-dimensional financial market model as the chartists' behaviour changes. The numerical analysis of our speculative market model is largely motivated by the work of Arnold et al. [2] and Schenk-Hoppé [14] on the noisy Duffing–van der Pol oscillator. To provide a complete picture of the market equilibrium behaviour of the model as a parameter capturing the chartists behaviour changes, we conduct our analysis from the viewpoints of both dynamical and phenomenological bifurcations. The so-called *dynamical* (D)-bifurcation examines the evolution of an initial point forward and backward in time and captures all the stochastic dynamics of the SDEs, while the so-called *phenomenological* (P)-bifurcation studies a stationary measure corresponding to the one-point motion. As indicated in Schenk-Hoppé [14] and the references cited there, the difference between P-bifurcation and D-bifurcation lies in the fact that the P-bifurcation approach focuses on long-run probability distributions, while the D-bifurcation approach is based on the invariant measure, the multiplicative ergodic theorem, the Lyapunov exponents and the occurrence of new invariant measures. However, the P-bifurcation has the advantage of allowing one to visualize the changes of the stationary density functions. Our results show that, as the chartists change their behaviour (through their extrapolation of the price trend), the market price can display different forms of equilibrium distribution. In particular, when chartists are less active, the market has a unique equilibrium distribution which is stable. However, when the chartists become more active, a new equilibrium distribution will be generated and become stable whilst the original equilibrium distribution becomes unstable. The corresponding stationary density will change from a single peak to a crater-like density. The market price can be driven away from the fundamental price. The change of stationary distribution is characterized by a bimodal logarithm price distribution and fat tails.

The structure of the paper is as follows. In Section 2, we first outline the extended model of Beja and Goldman [3] and Chiarella [5] with a stochastic fundamental price. In Sections 3 and 4, the stochastic dynamical behaviour is analyzed from the viewpoints of invariant measures and stationary measures respectively. The paper is then concluded in Section 5.

2. The model

Consider a financial market which consists of two types of investors, *fundamentalists* and *chartists* and two types of assets, a *risky* asset (e.g. a stock market index) and a *riskless* asset (e.g. a government bond). In the market, the transactions and price adjustments occur simultaneously. The changes of the risky asset price $P(t)$ are brought about by aggregate excess demand of fundamentalists (D_t^f) and chartists (D_t^c) at a finite speed of price adjustment. Accordingly, these assumptions may be expressed as

$$\mathrm{d}p(t) = [D_t^f + D_t^c]\mathrm{d}t, \tag{2.1}$$

where $p_t = \ln P(t)$ is the logarithm of the risky asset price $P(t)$ at time t.

The excess demand of the fundamentalists is assumed to depend on the market price deviation from the fundamental price, so that $D_t^f(p(t)) = a[F(t) - p(t)]$, where $a > 0$ and $F(t)$ denotes the logarithm of the fundamental price that clears fundamental demand at time t, that is $D_t^f(F(t)) = 0$. For the fundamental price $F(t)$ – in accordance with the theory of equilibrium prices in perfect markets – the successive changes in equilibrium value must be statistically independent. This proposition is usually formalized by the statement that $F(t)$ follows a random walk. Using the notation of stochastic differential equations, the fundamental value $F(t)$ can be considered to follow $\mathrm{d}F = \sigma \circ \mathrm{d}W$,