

# Time and scale Hurst exponent analysis for financial markets

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## Abstract

We use a new method of studying the Hurst exponent with time and scale dependency. This new approach allows us to recover the major events affecting worldwide markets (such as the September 11th terrorist attack) and analyze the way those effects propagate through the different scales. The time–scale dependence of the referred measures demonstrates the relevance of entropy measures in distinguishing the several characteristics of market indices: “effects” include early awareness, patterns of evolution as well as comparative behaviour distinctions in emergent/established markets.

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## 1. Introduction

The main goal of this study is the analysis of stock exchange world indices searching for signs of coherence and/or synchronization across the set of studied markets.

We have expanded the scope of previous work on the PSI-20 (Portuguese Standard Index), since results there [7] seemed to provide a basis for a wider ranging study of coherence and entropy.

With that purpose we applied econophysics techniques related to measures of “disorder”/complexity (entropy) and a newly proposed [8] generalisation of Detrended Fluctuation Analysis. As a measure of coherence among a selected set of markets we have studied the eigenvalues of the correlation matrices for two different set of markets [11], exploring the dichotomy represented by emerging and mature markets and proposing a more refined classification. The indices are used to represent or characterise the respective market.

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The classification of markets into mature or emergent is not a simple issue. The International Finance Corporation (IFC) uses per capita income and market capitalisation relative to Gross National Product (GNP) for classifying equity markets. If either (i) a market resides in a low or middle-income economy, or (ii) the ratio of the investable market capitalisation to GNP is low, then the IFC classifies the market as emerging, otherwise the classification is mature.

The data used in this study was taken daily for a set of worldwide market indices. As is usual in this kind of analysis [5] we base our results on the study of log returns  $\eta_i = \log \frac{x_i}{x_{i-1}}$ , where  $\eta_i$  is the log return at time step  $i$ .

## 2. Time and Scale Hurst exponent

### 2.1. Fractional Brownian motion

Fractional Brownian motion (fBm) is a well-known stochastic process where the second-order moments of the increments scale as

$$E\{(X(t_2) - X(t_1))^2\} \propto |t_2 - t_1|^{2H} \tag{1}$$

with  $H \in [0, 1]$ . The Brownian motion is then the particular case where  $H = 1/2$ .

The exponent  $H$  is called the Hurst exponent. If  $H < 1/2$ , then the behaviour is anti-persistent (intermediate memory), that is, deviations of one sign are generally followed by deviations with the opposite sign. The limiting case  $H = 0$ , (short memory), corresponds to white noise, where fluctuations at all frequencies are equally present.

If  $H > 1/2$ , then the behaviour is persistent (long memory), i.e. deviations tend to keep the same sign. The limiting case  $H = 1$ , reflects  $X(t) \propto t$ , a smooth signal.

While motivation for using fBm was the fat-tail characteristic of real price distributions, this  $H$ -threshold for persistent/anti-persistent behaviour is useful in terms of determining when trends break down. For emergent markets, value of  $H$  is consistently higher than 0.5 [11].

### 2.2. Detrended fluctuation analysis

The DFA (Detrended Fluctuation Analysis) technique consists in dividing a random variable sequence  $X(t)$ , of length  $s$ , into  $s/\tau$  non-overlapping boxes, each containing  $\tau$  points [9]. The linear local trend  $z(t) = at + b$  in each box is defined to be the standard linear least-square fit of the data points in that box. The detrended fluctuation function  $F$  is then defined by:

$$F_k^2(\tau) = \frac{1}{\tau} \sum_{t=k\tau+1}^{(k+1)\tau} |X(t) - z(t)|^2, \quad k = 0, \dots, \frac{s}{\tau} - 1. \tag{2}$$

Averaging  $F(\tau)$  over the  $s/\tau$  intervals gives the fluctuation  $\langle F(\tau) \rangle$  as a function of  $\tau$ . Here

$$\langle F^2(\tau) \rangle = \frac{\tau}{s} \sum_{k=0}^{s/\tau-1} F_k^2(t).$$

If the observable  $X(t)$  are random uncorrelated variables or short-range correlated variables, the behaviour is expected to be a power law

$$\sqrt{\langle F^2(\tau) \rangle} \sim \tau^H. \tag{3}$$

### 2.3. Method characterisation

When we apply the DFA to a time series we compute a single real number (the estimated Hurst exponent) that describes the global behaviour. One of the possible generalisations is to evaluate the Hurst exponent for fixed size windows, thus measuring local Hurst exponents and from there studying their time dependency (see Refs. [3,1]).

The general idea behind this method is the study of the local Hurst exponent as a function of both time and scale. In practical terms this method is a direct expansion of the “windowed” DFA applied in Ref. [7]. Instead of fixing  $s$  we

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