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Cluster behavior of a simple model in financial markets

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Abstract

We investigate the cluster behavior of financial markets within the framework of a model based on a scale-free network. In this model, a cluster is formed by connected agents that are in the same state. The cumulative distribution of clusters is found to be a power-law. We find that the probability distribution of the liquidity parameter, which measures the financial markets' energy, is rather robust. Furthermore, the time series of the liquidity parameter have the characteristics of $1/f$ noise, which may indicate the fractal geometry of financial markets.

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1. Introduction

Co-interaction and evolution between different agents are known to be one of the ingredients of complex systems, such as—social, biological, economical and technological systems. Following the trend of research on complex systems, to find the universal rules and principles of these systems become more and more attractive [1–6]. In particular, the studies of financial markets prices have been found to suggest several generalized properties similar to those observed in physical systems with a large number of interacting ingredients. More and more models have been introduced to attempt to capture the universalities behind the financial markets which are the so-called stylized facts [7–9], such as sharp peaks and fat-tail distributions for the financial prices, absence of autocorrelation in return, and long-time correlations in absolute return, etc. These models include the herding multi-agent model [10–14], the related percolation model [15,16] and the dynamic games model [17–22], etc.

Among the more sophisticated approaches are the multi-agent models, based on the interactions of two different agent groups ("noise" and "fundamental" traders), which reproduce some of the stylized facts of real markets but do not account for the origin of the universal characteristics. An alternative approach, the herd behavior [23] may be capable to induce the power-law asymptotic behavior in the tail of return distribution as found in the real data. But an

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assumption made in its model that should not be ignored is that the probability of each cluster to sell or buy is set to be the same and remains constant throughout the whole process, which may be a good strategy for simplifying a physical model but may not be a good regulation for establishing a model which we expect to reflect the various phenomena found in the real financial markets as genuine as possible. Here we introduce a model where the probability of each agent to sell or buy varies along with the difference between the demand and supply at each time step, which may be more helpful for us to learn the nature of the markets.

It is interesting to find that the various networks of the real-world, from social networks to biological networks, display scale-free degree distributions and small-world characteristics. So recently more and more models of financial markets have been proposed based on different types of networks [24–26]. It could characterize quantitatively the interaction between agents by means of a series of topological quantities, which could better capture the complex properties of the real-world. In Refs. [23,24,27], the models are in the view of the regular lattices and the famous Cont and Bouchaud model's network structure is that of the random graph. However, in our model, we consider a different topology on the scale-free network. More importantly, we obtain some interesting results about cluster behavior which is a very common phenomena in the real-world financial market.

This paper is organized as follows. In Section 2, we introduce our model. In Section 3, there are numerical simulations and some results. In Section 4, a discussion and main conclusions are given.

2. Our model

We present a model for the cluster formation and information dispersal based on a scale-free network. As a first approach to model the complicated social behaviors we consider: (1) the probability of each agent to sell or buy changes with the difference between the demand and supply of the markets, (2) agents having the same state and being linked form a cluster that makes consensual decisions, and (3) whenever a cluster forms, the information disperses instantly and each agent within the cluster randomly changes its state according to the difference between the demand and supply of the market. We then apply the model to studying the price dynamics in a financial market. In the present work, we consider a system of N agents, represented by N nodes in a scale-free network. The optional state of the agent i is represented by $\varphi_i = \{1, 0, -1\}$ corresponding to an inactive, waiting state ($\varphi_i = 0$), and two active states of either buying ($\varphi_i = 1$) or selling ($\varphi_i = -1$).

In order to mimic the scale-free network topology, we make use of the Barabási–Albert model [28] which is based on two main assumptions: (1) linear growth and (2) preferential attachment. The network is initialized with m_0 nodes. At each step a new node with m edges is added to the pre-existing network. The probability of preferential attachment that an edge of the new node is linked with the ith node is expressed by $\prod(k_i) = k_i / \sum_j k_j$ (k_i is the degree of node i).

A cluster is formed by all connected agents that have the same states. The size s_i of cluster i is defined as the total number of agents in the cluster. Initially, the states of all agents are randomly determined. Each cluster buys or sells a unit of financial product such as stock, future or currency at each time step with the same probability p, or waits with probability $1 - 2p$. Here, we call p the liquidity parameter. A value of $p < 0.5$ allows for a finite fraction of agents not to trade during a given time window. Our model is defined in the following way. At each time step t:

(1) the difference between the supply and demand of the markets is defined by:

$$d = \sum_{i=1}^{a} \varphi_i, \tag{1}$$

where a is the total number of clusters in the financial markets.

(2) the evolution of the probability of activity follows the rule:

$$p(t) = p(t-1)\mathrm{e}^{d/v}, \tag{2}$$

where v is a parameter that controls the update of the cluster size and provides a measure of the liquidity of the financial markets.

(3) the evolution of the financial index price follows the rule:

$$P(t) = P(t-1)\mathrm{e}^{d/z}, \tag{3}$$