Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 331 (2004) 233–239
PHYSICA A
www.elsevier.com/locate/physa

Arbitrage risk induced by transaction costs

Edward W. Piotrowski[a,*], Jan Sładkowski[b]

[a] *Institute of Theoretical Physics, University of Białystok, Lipowa 41, Białystok Pl 15424, Poland*
[b] *Institute of Physics, University of Silesia, Uniwersytecka 4, Katowice Pl 40007, Poland*

Received 2 June 2003

Abstract

We discuss the time evolution of quotation of stocks and commodities and show that they form an Ising chain. We show that transaction costs induce arbitrage risk that is usually neglected. The full analysis of the portfolio theory is computationally complex but the latest development in quantum computation theory suggests that such a task can be performed on quantum computers.
© 2003 Elsevier B.V. All rights reserved.

PACS: 02.50.Le; 03.67.Lx; 05.50.+q; 05.30.−d

Keywords: Econophysics; Financial markets; Quantum computations; Portfolio theory

1. Introduction

One can simply define arbitrage as an opportunity of making profit without any risk [1]. But this definition has one flaw: it neglects transaction costs. And any market activity involves costs (e.g. brokerage, taxes and others depending on the established rules). Therefore, there is always some uncertainty and an arbitrageur cannot avoid risk. Below we will describe an extremely profitable manipulation of a one asset market that certainly fits this definition and show how brokerage can induce risk. The method allows to make maximal profits in a fixed interval $[0,k]$ (short-selling allows to make profits with arbitrary price changes). We will analyze the associated risk by introducing *canonical arbitrage portfolios* that admit Ising model like description. Investigation of such models is difficult from the computational point of view (the complication grows

* Corresponding author.
E-mail addresses: ep@alpha.uwb.edu.pl (E.W. Piotrowski), sladk@us.edu.pl (J. Sładkowski).

0378-4371/$ - see front matter © 2003 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2003.09.015

exponentially in k) but the latest development in quantum computation seems to pave the way for finding effective methods of solving the involved computational problems [2].

2. Profit from brokerage-free transactions

Standard descriptions of price movements, following Bachelier, use the formalism of diffusion theory and random variables. Such an approach to the problem involves the assumption of constancy of the parameters of model during the interval used for their estimation. Besides the use of the dispersion of the drifting logarithm of price as a measure of the risk might be questioned. A clairvoyant that knows the future evolution of prices would make profits from any price movement and it would be difficult to attribute any risk to her market activity. Rather, the level of erroneousness of our decisions concerning the portfolio structure should be used for that aim. Having this in mind we have proposed a dual formulation of the portfolio theory and market prices [3–6]. In this approach, movements in prices are regarded as deterministic according to their historical record and the stochastic properties are attributed to portfolios. This enables us to use the formalism of information theory and thermodynamics. Due to the convenience of this approach we will adopt it in the current paper.

Consider a game against the Rest of the World (that is the whole market) that consist in alternate buying and selling of the same commodity. Let $h_m := \ln[c_m/(c_{m-1})]$ denote logarithms of the prices dictated by the market of this commodity at successive moments $m = 1, 2, \ldots, k$. If the costs of transactions are zero (or negligible) then the player's profit (actually a loss because for future convenience we will fix the sign in (1) according to the standard physical convention) in the interval $[0, k]$ is given by

$$H(n_1, \ldots, n_k) := -\sum_{m=1}^{k} h_m n_m \ . \tag{1}$$

The elements of the sequence (n_m) take the value 0 or 1 if the player possesses money or the commodity at the moment m, respectively. The sequence (n_m) defines the player's strategy in a unique way and any $(n_1, \ldots, n_k)$ describes a pure strategy. Generalization to a more realistic situation where more commodities are available is trivial but besides complication of formulas is irrelevant to the conclusion and will not be considered here.

3. The thermodynamics of portfolios

Any mixed strategy can be parameterized in a unique way by 2^k weights $p_{n_1,\ldots,n_k}$ giving the contributions of pure strategies. Let us consider as equivalent all strategies that for a given price sequence $(h_1, \ldots, h_k)$ bring the same profit. We will call the equivalence classes of portfolios defined in this way *the canonical portfolios*. Any

دریافت فوری ← متن کامل مقاله

ISIArticles
مرجع مقالات تخصصی ایران

✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
✓ امکان دانلود نسخه ترجمه شده مقالات
✓ پذیرش سفارش ترجمه تخصصی
✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
✓ دانلود فوری مقاله پس از پرداخت آنلاین
✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات