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Hedging options under transaction costs and stochastic volatility☆

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Abstract

In this paper, we consider the problem of hedging contingent claims on a stock under transaction costs and stochastic volatility. Extensive research has clearly demonstrated that the volatility of most stocks is not constant over time. As small changes of the volatility can have a major impact on the value of contingent claims, hedging strategies should try to eliminate this volatility risk. We propose a stochastic optimization model for hedging contingent claims that takes into account the effects of stochastic volatility, transaction costs and trading restrictions. Simulation results show that our approach could improve performance considerably compared to traditional hedging strategies. © 2002 Published by Elsevier Science B.V.

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1. Introduction

In this paper, we consider the problem of hedging contingent claims under transaction costs and stochastic volatility. Extensive research during the last two decades has demonstrated that the volatility of stocks is not constant over time (Bollerslev et al., 1992). Engle (1982) and Bollerslev (1986) introduced the family of ARCH and GARCH models to describe the evolution of the volatility of the asset price in discrete

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time. Econometric tests of these model clearly reject the hypothesis of constant volatility and find evidence of volatility clustering over time. In the financial literature stochastic volatility models have been proposed to model these effects in a continuous-time setting (Hull and White, 1987; Scott, 1987; Wiggins, 1987). Pricing methods for options on a stock with a stochastic volatility process are now widely available, both in the discrete-time and the continuous-time framework (Heston, 1993; Finucane and Tomas, 1997; Ritchken and Trevor, 1999). Practicable methods for hedging options under stochastic volatility are rare however.

Schweizer (1991, 1995) has proposed methods to minimize the replication error of contingent claims in general incomplete markets, including stochastic volatility as a special case. Schweizer (1995) only considers trading strategies involving the riskless bond and the underlying stock itself. As the bond and the underlying stock price are insensitive to changes of the volatility, these hedging schemes are deemed to be inefficient compared to strategies involving traded option contracts on the underlying stock (Frey and Sin, 1999). Traded options on the underlying stock are sensitive to changes in the stock price volatility. This observation is used in the simple delta–vega hedging scheme: traded options are added to the portfolio of the investor in order to eliminate the exposure to small changes of the volatility. Unfortunately, an effective delta–vega hedge has to be rebalanced frequently. As the bid–ask spreads on exchange-traded options are considerable, frequent updating of a delta–vega hedge could result in losses due to transaction costs.

Static hedging methods try to compose a buy-and-hold portfolio of exchange-traded options that replicate the payoff of the contingent claim under consideration (Derman et al., 1995; Carr et al., 1998). The static hedging strategy does not require any rebalancing and is therefore quite efficient in avoiding transaction costs. Unfortunately, the odds of coming up with a perfect static hedge for a particular over-the-counter product are small, as the number of (liquid) traded option contracts is limited. Avellaneda and Paras (1996) proposes an algorithm to construct a static portfolio of options that matches the desired payoff as closely as possible, while the residual is priced and hedged with a trading strategy involving the underlying stock. A disadvantage of this approach is that the static hedge can only be efficient if traded options are available with sufficiently similar maturity and moneyness as the over-the-counter product that has to be hedged.

In this paper, we propose a stochastic optimization model to extend the simple delta–vega hedging scheme. The hedge portfolio in our model consists of the underlying stock and exchange-traded options with sufficient liquidity. The model has a limited number of trading dates on which the hedge portfolio can be rebalanced (e.g. weekly), while transaction costs and trading restrictions are taken into account. The goal of the model is to minimize hedging errors by following an appropriate dynamic trading strategy. An important feature is that we only minimize the hedging error at the first few trading dates and not until the final maturity of the contingent claim. We think that our specification of the hedging model is useful because: (1) The planning horizon of traders is shorter than the maturity of their contingent claims and they are usually more interested in overnight profits and losses. (2) The portfolio of liabilities of a trader might change frequently due to additional buying and