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Research article

# Optimal controller synthesis for second order time delay systems with at least one RHP pole

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ABSTRACT

An optimal $H_2$ minimization framework is proposed in this paper for devising a controller of PID in nature, based on a refined IMC filter configuration. The tuning strategy is for controlling time delay system with at least one pole which falls on the right half of the s-plane. An underdamped model based filter is used in place of the unity damping ratio (critically damped) filter available in the literature to improve the reset action. The method has a single adjustable closed loop tuning parameter. Guidelines have been provided for choosing the pertinent tuning parameter based on the sensitivity function. Simulation work has been executed on diverse unstable models to support the advantages of the proposed scheme. The proposed controller yields improved performances over other recently reported tuning techniques in the literature. Experimental implementation is carried out on an inverted pendulum for demonstrating the practical applicability of the present method. The efficacy of the intended controller design is quantitatively analyzed using the time integral performance index.



## 1. Introduction

The Proportional, Integral and Derivative (PID) controller is the most used in industry despite the development of advanced control techniques. With the ease of operation and robust behaviour these controllers dominate use in the process industries. As these traditional controllers are quite adequate for all industrial processes which possess stable characteristics, they can also be put to practical use to tune the unstable systems that arise in many chemical and biological processes like polymerization reactors, bioreactors and continuous stirred tank reactors, etc. [1,2]. When such systems are modelled, the corresponding transfer function models have at least one pole located in the right half plane (RHP) and are called as unstable systems. Obviously, the tuning of these controllers for unstable processes to impart stability and good regulatory response is crucial and the difficulty increases in the presence of time delay. The existence of unstable systems is well described by Sree and Chidambaram [3]. In unstable systems, controlling a first order system is comparatively easier than that of a second order one (system with two unstable poles) as the order of the system is higher and there will be existence of process uncertainties in the two time constants [4–6]. There exist many PID tuning techniques for controlling unstable first order systems [7–11]. However, very few methods are available for control of unstable second order systems, viz., Rao and Chidambaram [6], Shamsuzzoha and Lee [1,12], Panda [13] and Cho et al. [14]. Modified Smith predictor based control schemes [15,16] and modified Internal Model Control (IMC) based control schemes [17] are also developed for unstable first order and second order models. However, these control strategies have more than one controller and tuning of those controllers is comparatively difficult [1,12]. Therefore, a simple robust controller providing good nominal output and control action responses may be preferable when compared to control schemes consisting of more than one controller. Hence, in this work, a novel methodology to enhance the closed loop performance of unstable second order plus time delay (USOPTD) processes is presented. Efforts have been put forth to develop a controller of PID type using IMC - $H_2$ minimization theory for second order unstable systems [2,5,9]. The distinctive feature of this approach is the presence of a single adjustable closed loop tuning parameter which vividly solves the performance-robustness trade-off aiming for good regulatory and servo responses. The proposed method deals with the design of PID controller by applying the IMC - $H_2$ minimization theory using an underdamped IMC filter. A set-point filter is considered for

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reducing the overshoot. Once the controller is designed, a detailed study is carried out for choosing the IMC parameter using robustness theory [18]. For clear illustration, the design is addressed in the next section followed by tuning parameter guidelines, simulation results and conclusions.

## 2. Controller design procedure

Consider the SISO (single input and single output) process transfer function $G_p$, along with the controller $G_c$ and setpoint filter $F_R$ as shown in Fig. 1.

Here $r, d$ and $y$ are the set point, disturbance and process output. In the proposed work, as a generalization, the controller structure is addressed for a process of order 2 having time delay with one or two right half plane (RHP) poles as given in Eq. (1) and can be rewritten as in Eq. (2).

$$G_p(s) = \frac{-k_q e^{-\theta s}}{(\tau_1 s - 1)(-\tau_2 s + 1)} \tag{1}$$

The IMC closed loop framework is depicted in Fig. 2. Here, $G_p(s)$ is the process, $G_m(s)$ is the equivalent transfer function model of $G_p(s)$ and the internal model controller is $Q_C$.

According to the concept of IMC

$$Q_C = \tilde{Q}_C F \tag{2}$$

where F(s) is the IMC filter which decides the robustness of the controller. In order to make the IMC controller $Q_c$ to be proper and realizable, the filter structure needs to be properly selected so that the internal control structure meets the stability conditions. Along with these requirements, the resulting controller is required to provide enhanced closed loop output responses.

To provide $H_2$ optimal performance, $\tilde{Q}_c$ is designed to obtain the $H_2$ optimal performance for a specific input type, v(s), where 'v' may be the set-point (r) or the disturbance (d). If the design is addressed by considering the disturbance, then v becomes equivalent to 'd'. Alternatively, if the design is addressed for a set-point change, v is equivalent to '−r' as defined by Morari and Zafiriou [19]. In general, 'v' can be a set point change or disturbance change affecting the process output such as steps, ramps, steps entering the first order lag, and so on. The model of the process and the specific step disturbance acting at the process input are separated as

$$G_m = G_{m-} G_{m+} \text{ and } v = v_- v_+ \tag{3}$$

'− ' denotes the invertible portion and ' + ' denotes the non -invertible portion, where the $G_{m+}$, $v_+$ are the non-minimum phase (NMP) elements which includes RHP zeros, time delays in the process and $G_{m-}$, $v_-$ are the minimum phase (MP) elements of the process and input disturbance which includes left half plane (LHP) zeros, process gain and time constants of the process model.

The Blaschke product of poles of Gm and v, are written as

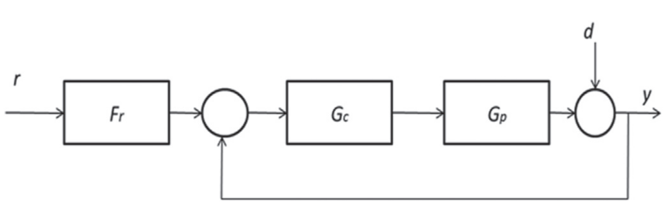


**Fig. 1.** Feedback Controller with a Set point filter.

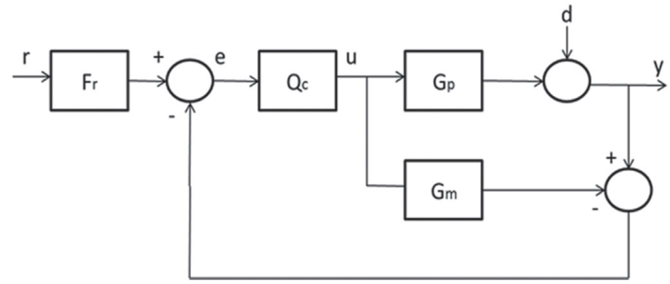


**Fig. 2.** IMC control scheme.

$$\begin{aligned} b_m(s) &= \prod_{i=1}^{k} (-s + p_{mi})/(s + \bar{p}_{mi}) \\ b_v(s) &= \prod_{i=1}^{k} (-s + p_{vi})/(s + \bar{p}_{vi}) \end{aligned} \tag{4}$$

where $p_i$ and $\bar{p}_i$ are the ith RHP pole and its conjugate correspondingly. Using the above terms, the optimal controller $\tilde{Q}_C$ is derived by minimizing the objective function [19].

$$e_2^2 = \int_0^\infty e^2 dt = \left\| (1 - G_m \tilde{Q}_c) v \right\|_2^2 \tag{5}$$

and the $H_2$ optimal controller is obtained as

$$\tilde{Q}_C = b_m (G_{m-} b_v v_-)^{-1} \left\{ (b_m G_{m+})^{-1} b_v v_- \right\}|_* \tag{6}$$

where $\{\ldots\}|_*$ denotes that after a partial fraction expansion of the operand all terms involving the poles of $(G_{m+})^{-1}$ are omitted. Nasution et al. [9] has applied this concept and has derived the IMC - PID controller. In this paper, this methodology is applied for second order process having unstable characteristics. By taking into account, the ideal model state, i.e., Gp = Gm, initially, the model of the process and the specific input are divided as MP and NMP elements.

$$G_{m^-} = \frac{k_q}{\tau_1 \tau_2(-s + (1/\tau_1))(-s + (1/\tau_2))} \text{ and } G_{m+} = e^{-\theta s} \tag{7}$$

$$v_{m^-} = \frac{k_q}{\tau_1 \tau_2 s\,(-s + (1/\tau_1))(-s + (1/\tau_2))} \text{ and } v_{m+} = 1 \tag{8}$$

Then, the Blaschke product is obtained as

$$\begin{aligned} b_m &= \frac{(-s + (1/\tau_1))\,(-s + (1/\tau_2))}{(s + (1/\tau_1))(s + (1/\tau_2))} \text{ and} \\ b_v &= \frac{(-s + (1/\tau_1))\,(-s + (1/\tau_2))}{(s + (1/\tau_1))(s + (1/\tau_2))} \end{aligned} \tag{9}$$

On the substitution of all expressions in Eq. (6), one can obtain,

$$\tilde{Q}_c = \frac{\left(-s - \frac{1}{\tau_1}\right)^2 \left(-s - \frac{1}{\tau_2}\right)^2 s \tau_1^2 \tau_2^2}{k_q^2} \left\{ \frac{k_q}{\tau_1 \tau_2 s \left(-s - \frac{1}{\tau_1}\right)\left(-s - \frac{1}{\tau_2}\right)} \right\}\Bigg|_* \tag{10}$$