AC OPF in radial distribution networks – Part II: An augmented Lagrangian-based OPF algorithm, distributable via primal decomposition

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ABSTRACT

In the first part of this two-part paper we show that the branch-flow convexification of the OPF problem is not exact and that the ADMM-based decomposition of the OPF fails to converge in specific scenarios. Therefore, there is a need to develop algorithms for the solution of the non-approximated OPF problem that remains inherently non-convex. To overcome the limitations of recent approaches for the solution of the OPF problem, we propose in this paper, a specific algorithm for the solution of a non-approximated, non-convex AC OPF problem in radial distribution systems. It is based on the method of multipliers, as well as on a primal decomposition of the OPF problem. We provide a centralized version, as well as a distributed asynchronous version of the algorithm. We show that the centralized OPF algorithm converges to a local minimum of the global OPF problem and that the distributed version of the algorithm converges to the same solution as the centralized one. Here, in this second part of the two-part paper, we provide the formulation of the proposed algorithm and we evaluate its performance by using both small-scale electrical networks, as well as a modified IEEE 13-node test feeder.

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1. Introduction

In Part I of this two-part paper we present the generic formulation of the non-convex OPF problem and we briefly review several OPF algorithms that are based on approximations and assumptions in order to guarantee convergence. Furthermore, we focus on the branch-flow convexification of the OPF problem that has been recently proposed by Farivar and Low in [1,2] as a form of exact solution for the case of radial distribution systems under specific assumptions. We show that that for such a property to hold, one needs to develop algorithms for the solution of the non-approximated OPF problem that remains inherently non-convex. In detail, we show through practical examples that in [1,2], on one hand, there is a misinterpretation of the physical network model related to the ampacity constraint of the lines’ current flows and, on the other hand, the proof of the exactness of the proposed relaxation requires unrealistic assumptions related to the unboundedness of specific control variables. Furthermore, we investigate the application of ADMM for the solution of the original non-approximated OPF problem. Even though ADMM requires the underlying problem to be convex in order to guarantee convergence, it was applied also to the case of non-convex AC OPF problems with promising convergence performance (e.g., [3,4]). However, we show, through practical examples, cases for which the ADMM-based decomposition of the non-relaxed OPF problem fails to converge.

To overcome the aforementioned limitations, here in this second part, we propose an algorithm for the solution of the non-approximated non-convex AC OPF problem in radial networks. A large number of algorithms can be found in the literature that tackle the non-approximated non-convex OPF problem ranging from non-linear and quadratic programming techniques, Newton-based methods, interior point methods to heuristic approaches based on genetic algorithms, evolutionary programming, and particle-swarm optimization (e.g., [5–8]). These methods, typically utilize
powerful general purpose solvers or in-house developed software but they do not provide, in general, convergence guarantees. Our proposed solution belongs to the family of augmented Lagrangian methods for the solution of the OPF problem. More specifically, our proposed method uses an augmented Lagrangian approach, relies on the method of multipliers [9,11] and provides convergence guarantees. In particular, we design a centralized OPF algorithm that is proven to converge to a local minimum of the original non-approximated OPF problem.

With respect to the case of controlling multiple dispersed energy resources, it is of interest to also define a distributed solution method that is formally equivalent to the centralized formulation. Distributed solutions are of interest in several practical cases. Among others, when the problem size is large due to a very large number of small controllable resources like PV panels, when the communication requirements are such that the amount of exchanged information flow between agents needs to be limited, or when an asynchronous solution of the problem is more appealing. In fact, several distributed OPF algorithms are proposed in the literature. In [12,13] the authors design a dual-ascent algorithm for optimal reactive power flow with power and voltage constraints. In [14,15] dual decomposition is used as the basis for the distributed solution of the OPF problem.

A further category of distributed OPF has been proposed to solve multi-area objectives [16,17]. However, their applicability to generically decomposable OPF has not been discussed. Furthermore, the proposed method in [16,17] consists essentially in finding a stationary point of the Lagrangian of the targeted OPF problem, which is a necessary condition for an optimum but might be not a sufficient one. Finally, a significant number of contributions propose distributed formulations of the OPF problem, based on the alternating direction method of multipliers (ADMM) (e.g., [18,14,3,19,20,4]).

In this direction, we further present here in this second part, a distributed version of the proposed algorithm that, unlike ADMM, is based on a primal decomposition [21] and does not require that the problem be convex. In this decentralized version of the algorithm, at each iteration, local agents, assigned to network buses and network lines, exchange messages with their neighbors using only local information. We prove that the distributed algorithm converges to the same solution as the centralized version. Finally, we present an asynchronous implementation of the distributed algorithm where the messages of the neighboring agents need not be synchronized.

The structure of this second part is the following. In Section 2 we describe the proposed algorithm for the OPF solution. We present both a centralized, as well as a decentralized asynchronous version of the proposed algorithm. In Section 3 we investigate the convergence of the proposed algorithm in the cases where the BFM convexification leads to an incorrect solution and ADMM fails to converge to a solution. In Section 4 we evaluate the performance of the proposed algorithm using a modified IEEE 13-node test feeder. Finally, in Section 5 we provide the main observations and concluding remarks for this Part II.

2. AC OPF in radial distribution systems

We first write the AC OPF problem presented in Part I in an equivalent form, and then we provide a centralized, as well as a distributed algorithm for its resolution.

We make the following assumptions about the grid model:

- Any two-port component (e.g., lines, transformers, etc.) is represented as a \( \pi \)-equivalent;
- We assume a perfect knowledge of the system parameters, i.e., the network admittance matrix is known;
- The nodal-power injections are voltage-independent;
- The control variables are composed by the nodal power injections/absorptions.

2.1. The proposed centralized OPF algorithm

We are interested in maximizing the social welfare of the economic agents that use the grid, while maintaining an acceptable network voltage profile and respecting the line ampacity limits. Specifically, we tune the line ampacities and the network voltage profiles by controlling the \((P,Q)\)-injections of distributed controllable devices \(G\) (e.g., renewable generators) in a “fair” way: Each controllable device \( g \in G \) has a certain utility function \( U_g(\cdot) \), and the sum of these utility functions is maximized subject to the satisfaction of the network operation constraints (voltage and ampacity). The resulting set-point is thus Pareto-optimal, i.e., no single device can increase its utility without hurting the utility of some other device, and locally—“fair”, i.e., the resulting set-point is a local maximizer of the sum of the device utilities lying on the Pareto boundary of feasible set-points.

By convention, each line \( \ell \in \mathcal{L} \) has a “receiving” and a “sending” end, which we denote by \( \ell^+ \) and \( \ell^- \), respectively. These are chosen arbitrarily. A line is connected to two adjacent buses to which we refer by \( B(\ell^+) \) and \( B(\ell^-) \), respectively. For each line, we introduce two auxiliary variables \( \bar{E}_{\ell^+} \) and \( \bar{E}_{\ell^-} \) representing the complex voltage at the two ends of the line. Assumptions A1–A3 allow us to express the corresponding injected currents and powers at the two ends of line \( \ell \):

\[
I_{\ell^+} = \bar{I}_{\ell^+}(\bar{E}_{\ell^+}, \bar{E}_{\ell^-}) = (Y_0 + Y_0) \bar{E}_{\ell^+} - Y_0 \bar{E}_{\ell^-} \\
I_{\ell^-} = \bar{I}_{\ell^-}(\bar{E}_{\ell^+}, \bar{E}_{\ell^-}) = (Y_0 + Y_0) \bar{E}_{\ell^-} - Y_0 \bar{E}_{\ell^+} \\
S_{\ell^+} = \bar{S}_{\ell^+}(\bar{E}_{\ell^+}, \bar{E}_{\ell^-}) = \bar{E}_{\ell^+} I_{\ell^+} \\
S_{\ell^-} = \bar{S}_{\ell^-}(\bar{E}_{\ell^+}, \bar{E}_{\ell^-}) = \bar{E}_{\ell^-} I_{\ell^-} 
\]

In the remainder of this paper, unless otherwise stated, the complex line currents and powers expressed above are always computed according to Eqs. (1)–(4). They are thus all functions of \( \bar{E}_{\ell^+} \) and \( \bar{E}_{\ell^-} \) exclusively, although the arguments are often omitted for the sake of brevity. All quantities are expressed in “per-unit”, unless otherwise specified.

For readability, we denote the vector formed by the real and imaginary parts of variables \( \bar{E}_{\ell^+} \) and \( \bar{E}_{\ell^-} \) by \( \bar{y} \in \mathbb{R}^{2|\mathcal{L}|} \), where \( |\mathcal{L}| \) is the number of lines. Note that for a given value of \( \bar{y} \), the corresponding currents and powers do not necessarily satisfy Kirchhoff’s law.

We call \( y \) feasible if it satisfies voltage consistency and per-bus power-balance. Voltage consistency means that the voltages of all the lines incident to a specific bus \( b \in \mathcal{B} \) are identical, i.e., have the same amplitude \( V_b \) and the same argument \( \varphi_b \):

\[
|\bar{E}_{\ell^+}| = V_{R(\ell^+)} , \ |\bar{E}_{\ell^-}| = V_{R(\ell^-)} \\
\arg(\bar{E}_{\ell^+}) = \varphi_{R(\ell^+)} , \ \arg(\bar{E}_{\ell^-}) = \varphi_{R(\ell^-)} \quad \forall \ell \in \mathcal{L}.
\]
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