Fair risk allocation in illiquid markets

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ABSTRACT

Let us consider a financial firm having some divisions which have invested into some risky assets and have some liabilities. As a cushion against possible future losses, the firm should hold some capital (equity), otherwise it would not be credible that it can pay back its liabilities. A coherent measure of risk (Artzner et al., 1999) assigns a number to the profit and loss distribution of the value of the portfolio of the firm at a specific future point of time. When gross assets (without netting the liabilities) are taken as the portfolio of the firm, then the measure of risk reflecting the preferences of the regulator or the firm is negative, its absolute value can be seen as a “safe valuation” of the assets. The capital requirement in this case can be calculated in the following way: liabilities minus the safe value of the assets determined by the measure of risk. Since the firm should keep capital in riskless assets, dividing the returns of the divisions by the respective capital requirements can serve, for instance, as a performance evaluation measure. Using coherent measures of risk the sum of the capital requirements of the divisions is larger than the capital requirement of the firm itself, there is some diversification benefit that should be allocated somehow.1

Csóka and Herings (2014) extend the usual cooperative game theory approach (risk allocation games) to handle the problem of risk capital allocation when the divisions might have illiquid assets by combining the notions of

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1 For more details and applications see Denault (2001); Kalkbrener (2005); Buch and Dorfler (2008); Homburg and Scherpereel (2008); Kim and Hardy (2009); Csóka et al. (2009); Balog et al. (2016) and Csóka and Pintér (2016) among others.

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Csóka et al. (2009) and Acerbi and Scandolo (2008). To define a cooperative game one should assign a payoff to all coalitions of players, that is, to all subsets of the divisions. In a risk allocation game, the payoff of a coalition is the opposite of its risk, where risk is measured by using a coherent measure of risk on the possible realizations of the value of the portfolio of the coalition. When having illiquid portfolios, the realization value of the portfolio of a coalition in a particular state depends on how easy it is to trade its assets (captured by random marginal demand curves) and on the required composition of the portfolio (called liquidity policy). To analyze how funding constraints affect fair risk allocation, in this paper we will use cash liquidity policies, where a certain amount of cash should be generated and short sales are not allowed. As a novelty, we add short sale constraints to the problem, since to cover the risk of upward moving prices, usually the proceeds of a short sale are not allowed to be used in another transaction. The random marginal demand curves lead to different optimal trades (sales) to satisfy the cash liquidity policy of the firm. After executing optimal trades by the coalition of divisions, the realized value of the resulting portfolio is determined by using the best bid prices of the resulting assets. Since the coalition of the divisions could trade at the same time, they face an externality problem: their optimal trade depends on the trades done by the other divisions outside the coalition. Csóka and Herings (2014) argue that the most reasonable way to handle this problem is that the divisions outside the coalition at hand remain inactive, so their portfolios can be considered as fixed and they define risk allocation games with liquidity this way.

Having the payoffs of the coalitions allows us to use standard game theory concepts (like the Shapley value (Shapley, 1953) or the nucleolus (Schmeidler, 1969)) as risk allocation rules to split up the risk capital of the firm to its divisions in any possible situations. In this paper, we will assess the possibility to jointly satisfy three fairness requirements for allocating risk capital in illiquid markets introduced by Csóka and Pintér (2016): Core Compatibility, Equal Treatment Property, and Strong Monotonicity. Core Compatibility is satisfied if the risk of the firm is allocated in such a way that no coalition of the divisions would have a lower risk by being alone. Such allocations are said to be in the core of the game. Csóka and Pintér (2016) notes that Core Compatibility can also be viewed as the allocated risk to each coalition of divisions should be at least as much as the risk increment the coalition causes by joining the rest of the divisions. Equal Treatment Property guarantees that if two divisions have the same stand-alone risk and also they contribute the same risk to all subsets of divisions not containing them, then the same risk capital should be allocated to them. Strong Monotonicity requires that if a division weakly reduces its stand-alone risk and also its risk contribution to all subsets of the other divisions, then, as an incentive, its allocated risk capital should not increase.

A subgame is obtained by considering a subset of the divisions of the firm and looking the resulting risk allocation game. A totally balanced game has a non-empty core in all of its subgames. Csóka and Herings (2014) show that when using coherent measures of risk, the class of risk allocation games with liquidity coincides with the class of totally balanced games, generalizing the result by Csóka et al. (2009) for risk allocation game without liquidity. The coincidence means that firstly, any totally balanced game can be generated by a suitably chosen risk allocation game with or without liquidity and secondly, it also means that any risk allocation game with or without liquidity is totally balanced, that is, Core Compatibility alone can be satisfied. Csóka and Pintér (2016) show that on the class of totally balanced games the Shapley value is the only risk allocation rule satisfying Equal Treatment Property and Strong Monotonicity at the same time. However, it is well-known that the Shapley value does not satisfy Core Compatibility in general, hence in theory the three requirements are irreconcilable.

Looking at the impossibility problem from a practical perspective, the Shapley value in a random but realistic risk allocation game with liquidity is not always expected to satisfy Core Compatibility. Hence we can assess the possibility to allocate risk in a fair way in illiquid markets by checking the average Core Compatibility of the Shapley value in such random risk environments with liquidity considerations. In the simulation, we will consider 3 to 8 divisions and in each treatment we simulate 10 000 random risk allocation games with liquidity. We observe that Core Compatibility is low (below 50%) and gets very low (below 10%) when increasing the number of divisions to 8. So we can say that it is practically not possible to allocate risk in illiquid markets satisfying the three fairness notions at the same time, one has to give up at least one of them. Both increasing the cash to be generated in the liquidity policy and raising the percentage of outcomes from which the Expected Shortfall is calculated increase Core Compatibility.

We are aware of two papers doing similar simulations. Homburg and Scherpereel (2008) simulate the average Core Compatibility of the Shapley value to be above 70% up to 10 divisions, but in their setting Value at Risk is used and there are no liquidity constraints. It is well-known that Value at Risk is not a coherent measure of risk, hence that result says nothing about the other two fairness requirements. Balog et al. (2016) also simulate random risk allocation games with coherent measures of risk, but without liquidity constraints.

The structure of the paper is as follows. In Section 2 we describe risk allocation games with liquidity constraints. Section 3 defines the Shapley value and discusses some of its main fairness properties. Section 4 contains the simulation results and Section 5 concludes.

2. Risk allocation games with liquidity constraints

We consider a firm with $n$ divisions, whose risk capital should be allocated. Risk environments with liquidity considerations are defined by Csóka and Herings (2014) and are denoted by $(N, J, S, \pi, \theta, m, L, \rho)$, where

- $N$ is the set of divisions,
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