

Adaptive Unscented Kalman Filter using Maximum Likelihood Estimation[★]

Zeinab Mahmoudi, Niels Kjølstad Poulsen, Henrik Madsen,
John Bagterp Jørgensen

*Department of Applied Mathematics and Computer Science, Technical
University of Denmark, 2800 Kgs. Lyngby, Denmark (e-mail:
jbjo@dtu.dk).*

Abstract: The purpose of this study is to develop an adaptive unscented Kalman filter (UKF) by tuning the measurement noise covariance. We use the maximum likelihood estimation (MLE) and the covariance matching (CM) method to estimate the noise covariance. The multi-step prediction errors generated by the UKF are used for covariance estimation by MLE and CM. Then we apply the two covariance estimation methods on an example application. In the example, we identify the covariance of the measurement noise for a continuous glucose monitoring (CGM) sensor. The sensor measures the subcutaneous glucose concentration for a type 1 diabetes patient. The root-mean square (RMS) error and the computation time are used to compare the performance of the two covariance estimation methods. The results indicate that as the prediction horizon expands, the RMS error for the MLE declines, while the error remains relatively large for the CM method. For larger prediction horizons, the MLE provides an estimate of the noise covariance that is less biased than the estimate by the CM method. The CM method is computationally less expensive though.

© 2017, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Unscented Kalman filter, Maximum likelihood estimation, Covariance matching technique, Adaptive filtering, Covariance estimation, Continuous glucose monitors.

1. INTRODUCTION

Identifying the uncertainties that affect a system is fundamental for monitoring and designing an optimal and adaptive estimator. In order to have a filter that is close to optimal, we need to know the covariance of the process and measurement noise. Methods for identifying noise covariances often include maximum likelihood estimation (MLE) (Zagrobelyny and Rawlings, 2015b; Jørgensen and Jørgensen, 2007), covariance matching (CM) techniques (Maybeck, 1982; Weige et al., 2015; Partovibakhsh and Liu, 2015), and correlation-based approaches such as the autocovariance least-squares (ALS) method (Åkesson et al., 2008; Odelson et al., 2006a,b; Zagrobelyny and Rawlings, 2015a). These methods often deal with only linear or linearized systems. The CM technique is commonly used for the nonlinear systems, because it is computationally inexpensive and it is flexible to accommodate the nonlinear models. It is a suboptimal covariance estimation though. The literature on the use of optimization-based covariance estimation approaches for nonlinear systems is sparse.

The purpose of this study is to use an optimization-based estimator, i.e., the MLE method, for identification of the noise covariance in a nonlinear system. Furthermore, we compare the MLE approach with a suboptimal estimation method, i.e., the CM algorithm, in the context of an adaptive unscented Kalman filter (UKF). We employ the

estimation of the measurement noise covariance as the basis for deriving filter adaptation.

The paper is structured as follows. First, we present the unscented Kalman filter (UKF) for prediction, filtering, and generating the covariance matrix of the prediction errors. Then, we develop the MLE problem and the CM technique to estimate the covariance of the measurement noise. The multi-step prediction errors and their covariances generated by the UKF are used in the MLE and CM methods. We then apply the MLE and CM algorithms on an example. The example is a nonlinear metabolic model of a patient with type 1 diabetes. In the example, we estimate the noise covariance of a continuous glucose monitoring (CGM) sensor, and derive the adaptive UKF to filter the sensor measurements.

2. MATERIALS AND METHODS

2.1 The unscented Kalman filter

In the UKF, a set of sigma points are deterministically chosen to represent the mean and covariance of the states. The sigma points therefore approximate the probability distribution of the states as it goes through the nonlinear transformation (Särkkä, 2007; Julier and Uhlmann, 2004). Approximating the probability distribution of the states by the sigma points in the UKF has shown to produce less estimation bias compared to the linearization in the extended Kalman filter (EKF) (Simon, 2006).

[★] This work is funded by the Danish Diabetes Academy supported by the Novo Nordisk Foundation.

Model The model of the state space in the stochastic differential equation (SDE) form and the measurement model are of the form

$$dx(t) = f(x(t), u(t), d(t)) dt + \sigma \cdot d\omega(t), \quad (1a)$$

$$y_k = g(x_k) + \xi_k, \quad (1b)$$

$$d\omega(t) \sim N_{iid}(0, Idt), \quad (1c)$$

in which x is the state, u is the input, d is the disturbance, and y is the measurement. We assume that ξ is a Gaussian zero-mean discrete-time measurement noise with covariance R . The stochastic noise ω is a standard Wiener process, and σ is the diffusion coefficient. I is an $n \times n$ identity matrix, where n is the number of state variables in the model.

Prediction This section explains the multi-step prediction with the UKF. The prediction steps are $j = 1, 2, \dots, N_p$ and N_p is the prediction horizon. The scaling parameter λ determines how far the sigma points are scattered away from the mean. λ and c are defined as

$$\lambda = \alpha^2(n + \kappa) - n, \quad c = \alpha^2(n + \kappa). \quad (2a)$$

The associated weights, W , for the $2n + 1$ sigma points are given by

$$W_m^{(0)} = \lambda / (n + \lambda), \quad (2b)$$

$$W_c^{(0)} = \lambda / \{(n + \lambda)(1 - \alpha^2 + \beta)\}, \quad (2c)$$

$$W_m^{(i)} = 1 / \{2(n + \lambda)\}, \quad i = 1, \dots, 2n \quad (2d)$$

$$W_c^{(i)} = 1 / \{2(n + \lambda)\}, \quad i = 1, \dots, 2n \quad (2e)$$

$$W_m = [W_m^{(0)} \dots W_m^{(2n)}]^T. \quad (2f)$$

A deterministic approach, based on the Cholesky factorization of the covariance P , samples the probability distribution to generate the sigma points \hat{X} .

$$\begin{aligned} \hat{X}_{k+j-1} &= [\hat{x}_{k+j-1|k} \dots \hat{x}_{k+j-1|k}] \\ &+ \sqrt{c} [0 \quad \sqrt{P_{k+j-1|k}} \quad -\sqrt{P_{k+j-1|k}}] \\ &= [m^{(i)} \dots m^{(2n)}], \quad i = 0, \dots, 2n. \end{aligned} \quad (3a)$$

The nonlinear function f propagates each of the sigma points according to

$$\frac{d\hat{X}_{k+j-1}}{dt}(t) = f(\hat{X}_{k+j-1}(t), u(t), d(t)), \quad t \in [t_{k+j-1} \ t_{k+j}] \quad (3b)$$

$$\hat{X}_{k+j} = \hat{X}_{k+j-1}(t_{k+j}). \quad (3c)$$

The parameters α , κ , and β are set to $\alpha = 0.01$, $\kappa = 0$, and $\beta = 2$. The weighted average of the transformed sigma points gives the predicted mean

$$\hat{x}_{k+j|k} = \hat{X}_{k+j} W_m. \quad (3d)$$

The covariance of the estimation error is computed by propagating dP_{k+j-1} according to

$$\begin{aligned} &\frac{dP_{k+j-1}(t)}{dt} \\ &= \sum_{i=0}^{2n} W_c^{(i)} \left(m^{(i)}(t) - m_x(t) \right) \left(f(m^{(i)}(t), u(t)) - m_f(t) \right)^T \\ &+ \sum_{i=0}^{2n} W_c^{(i)} \left(f(m^{(i)}(t), u(t)) - m_f(t) \right) \left((m^{(i)}(t) - m_x(t)) \right)^T \\ &+ \sigma \sigma^T, \quad t \in [t_{k+j-1} \ t_{k+j}] \end{aligned} \quad (3e)$$

where m_x and m_f are

$$\begin{aligned} m_x(t) &= \sum_{i=0}^{2n} W_m^{(i)} m^{(i)}(t), \\ m_f(t) &= \sum_{i=0}^{2n} W_m^{(i)} f(m^{(i)}(t), u(t)). \end{aligned} \quad (3f)$$

The propagated error covariance is then

$$P_{k+j|k} = P_{k+j-1}(t_{k+j}). \quad (3g)$$

To increase accuracy, new sigma points are generated from the predicted state mean and covariance as indicated in

$$\begin{aligned} \tilde{X}_{k+j} &= [\hat{x}_{k+j|k} \dots \hat{x}_{k+j|k}] \\ &+ \sqrt{c} [0 \quad \sqrt{P_{k+j|k}} \quad -\sqrt{P_{k+j|k}}] \\ &= [\tilde{m}^{(i)} \dots \tilde{m}^{(2n)}], \quad i = 0, \dots, 2n. \end{aligned} \quad (3h)$$

The measurement model transforms each of the new sigma points

$$\hat{Y}_{k+j} = g(\tilde{X}_{k+j}) = [\mu^{(i)} \dots \mu^{(2n)}], \quad i = 0, \dots, 2n. \quad (4a)$$

The weighted average of the measurement sigma points gives the predicted measurement

$$\hat{y}_{k+j|k} = \hat{Y}_{k+j} W_m, \quad (4b)$$

and the j -step prediction error is given by

$$e_{k+j|k} = y_{k+j} - \hat{y}_{k+j|k}. \quad (4c)$$

S_{k+j}^{yy} is the covariance of \hat{Y}_{k+j} and is computed as

$$S_{k+j}^{yy} = \sum_{i=0}^{2n} W_c^{(i)} \left(\mu^{(i)} - \hat{y}_{k+j|k} \right) \left(\mu^{(i)} - \hat{y}_{k+j|k} \right)^T. \quad (4d)$$

S_{k+j} is the covariance of $e_{k+j|k}$ and is calculated by

$$S_{k+j} = S_{k+j}^{yy} + R_{k+j}. \quad (4e)$$

Filtering The equation set (5) describes filtering and measurement update with the UKF. S_{k+1}^{xy} is the cross-covariance of \tilde{X} and $\hat{Y}_{k+j|k}$, and can be estimated as

$$S_{k+1}^{xy} = \sum_{i=0}^{2n} W_c^{(i)} \left(\tilde{m}^{(i)} - \hat{x}_{k+1|k} \right) \left(\mu^{(i)} - \hat{y}_{k+1|k} \right)^T, \quad (5a)$$

and K_{k+1} is the filter gain as follows

$$K_{k+1} = S_{k+1}^{xy} S_{k+1}^{-1}. \quad (5b)$$

The updated state mean is computed as

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_{k+1} - \hat{y}_{k+1|k}). \quad (5c)$$

The updated error covariance is given by

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1} S_{k+1} K_{k+1}^T. \quad (5d)$$

Multi-step prediction error and its covariance matrix Let $\{y_k\}_{k=1}^N$ denote the measurements and N_p denote the prediction horizon. Let the time indices be $k = 0, 1, \dots, N - N_p$, and the prediction index be $1 \leq j \leq N_p$. This implies that $1 \leq k + j \leq N$. Let ϵ_{k+N_p} denote the vector of the prediction errors in the N_p -sample prediction window as

$$\epsilon_{k+N_p} = \begin{bmatrix} e_{k+1|k} \\ e_{k+2|k} \\ \vdots \\ e_{k+N_p|k} \end{bmatrix} = \begin{bmatrix} y_{k+1} - \hat{y}_{k+1|k} \\ y_{k+2} - \hat{y}_{k+2|k} \\ \vdots \\ y_{k+N_p} - \hat{y}_{k+N_p|k} \end{bmatrix}. \quad (6)$$

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات