Adaptive Kalman Filter for Actuator Fault Diagnosis

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Abstract: An adaptive Kalman filter is proposed in this paper for actuator fault diagnosis in discrete time stochastic time varying systems. By modeling actuator faults as parameter changes, fault diagnosis is performed through joint state-parameter estimation in the considered stochastic framework. Under the classical uniform complete observability-controllability conditions and a persistent excitation condition, the exponential stability of the proposed adaptive Kalman filter is rigorously analyzed. The minimum variance property of the combined state and parameter estimation errors is also demonstrated. Numerical examples are presented to illustrate the performance of the proposed algorithm.

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1. INTRODUCTION

In order to improve the performance and the reliability of industrial systems, and to satisfy safety and environmental requirements, researches and developments in the field of fault detection and isolation (FDI) have been continuously progressing during the last decades (Hwang et al., 2010). Model-based FDI have been mostly studied for linear time invariant (LTI) systems (Gertler, 1998; Chen and Patton, 1999; Isermann, 2005; Ding, 2008), whereas nonlinear systems have been studied to a lesser extent and limited to some particular classes of systems (De Persis and Isidori, 2001; Xu and Zhang, 2004; Berdjaj et al., 2006). This paper is focused on actuator fault diagnosis for linear time-varying (LTV) systems, including the particular case of linear parameter varying (LPV) systems. The problem of fault diagnosis for a large class of nonlinear systems can be addressed through LTV/LPV reformulation and approximations (Lopes dos Santos et al., 2011; Tóth et al., 2011). It is thus an important advance in FDI by moving from LTI to LTV/LPV systems.

In this paper, actuator faults are modeled as parameter changes, and their diagnosis is achieved through joint estimation of states and parameters of the considered LTV/LPV systems. Usually the problem of joint state-parameter estimation is solved by recursive algorithms known as adaptive observers, which are most often studied in deterministic frameworks for continuous time systems (Marino and Tomei, 1995; Zhang, 2002; Besançon et al., 2006; Farza et al., 2014). Discrete time systems have been considered in (Guyader and Zhang, 2003; Ticlea and Besançon, 2016), also in deterministic frameworks. In order to take into account random uncertainties with a numerically efficient algorithm, this paper considers stochastic systems in discrete time, with an adaptive Kalman filter, which is structurally inspired by adaptive observers (Zhang, 2002; Guyader and Zhang, 2003), but with well-established stochastic properties.

The main contribution of this paper is an adaptive Kalman filter for discrete time LTV/LPV system joint state-parameter estimation in a stochastic framework, with rigorously proved stability and minimum variance properties.

Different adaptive Kalman filters have been studied in the literature for state estimation based on inaccurate state-space models. Most of these algorithms address the problem of unknown (or partly known) state noise covariance matrix or output noise covariance matrix (Mehra, 1970; Brown and Ratan, 1985), whereas the case of incorrect state dynamics model is treated as incorrect state covariance matrix. In contrast, in the present paper, the new adaptive Kalman filter is designed for actuator fault diagnosis, by jointly estimating states and parameter changes caused by actuator faults.

2. PROBLEM STATEMENT

The discrete time LTV system subject to actuator faults considered in this paper is generally in the form of

\[ x(k) = A(k)x(k-1) + B(k)u(k) + \Phi(k)\theta + w(k) \]  
\[ y(k) = C(k)x(k) + v(k) \]

where \( k = 0, 1, 2, \ldots \) is the discrete time instant index, \( x(k) \in \mathbb{R}^n \) is the state, \( u(k) \in \mathbb{R}^l \) the input, \( y(k) \in \mathbb{R}^m \) the output, \( A(k), B(k), C(k) \) are time-varying matrices of appropriate sizes characterizing the nominal state-space model, \( w(k) \in \mathbb{R}^m, v(k) \) are mutually independent centered white Gaussian noises of covariance matrices \( Q(k) \in \mathbb{R}^{n \times n} \) and \( R(k) \in \mathbb{R}^{m \times m} \), and the term \( \Phi(k)\theta \) represents actuator faults with a known matrix sequence \( \Phi(k) \in \mathbb{R}^{n \times p} \) and a constant (or piecewise constant with rare jumps) parameter vector \( \theta \in \mathbb{R}^p \).

A typical example of actuator faults represented by the term \( \Phi(k)\theta \) is actuator gain losses. When affected by such faults, the nominal control term \( B(k)u(k) \) becomes

\[ B(k)(I_l + \text{diag}(\theta))u(k) = B(k)u(k) - B(k)\text{diag}(u(k))\theta \]

There exists a “forward” variant form of the state-space model, typically with \( x(k + 1) = A(k)x(k) + B(k)u(k) + w(k) \). While this difference is important for control problems, it is not essential for estimation problems, like the one considered in this paper. The form chosen in this paper corresponds to the convention that data are collected at \( k = 1, 2, 3, \ldots \) and the initial state refers to \( x(0) \).
where $I_l$ is the $l \times l$ identity matrix, the diagonal matrix $\text{diag}(\theta)$ contains gain loss coefficients within the interval $[0, 1]$, and $\Phi(k) \in \mathbb{R}^{n \times l}$ ($p = l$) is, in this particular case,

$$\Phi(k) = -B(k)\text{diag}(u(k)).$$

A straightforward solution for the joint estimation of $x(k)$ and $\theta$ is to apply the Kalman filter to the augmented system

$$\begin{bmatrix}
    x(k) \\
    \theta(k)
\end{bmatrix} =
\begin{bmatrix}
    A(k) & \Phi(k) \\
    0 & I_p
\end{bmatrix}
\begin{bmatrix}
    x(k-1) \\
    \theta(k-1)
\end{bmatrix} +
\begin{bmatrix}
    B(k) \\
    0
\end{bmatrix} u(k) +
\begin{bmatrix}
    w(k) \\
    0
\end{bmatrix}
$$

$$y(k) = [C(k) \ 0]
\begin{bmatrix}
    x(k) \\
    \theta(k)
\end{bmatrix} + v(k).$$

However, to ensure the stability of the Kalman filter, this augmented system should be uniformly completely observable and uniformly completely controllable regarding the state noise (Kalman, 1963; Jazwinski, 1970). Notice that, even in the case of time invariant matrices $A$ and $C$, the augmented system is time varying because of $\Phi(k)$, which is typically time varying. The uniform complete observability of an LTV system is defined as the uniform positive definiteness of its observability Gramian (Kalman, 1963; Jazwinski, 1970). In practice, it is not natural to directly assume properties (observability and controllability) of the augmented system (a more exaggerated way would be directly assuming the stability of its Kalman filter, or anything else that should be proved!). Moreover, the augmented system is clearly not uniformly completely controllable regarding the state noise $w(k)$, since the augmented states $\theta(k)$ are not controlled at all by $w(k)$.

In contrast, in the present paper, the classical uniform complete observability and uniform complete controllability are assumed for the original system (1), in terms of the Gramian matrices defined for the $[A(k), C(k)]$ pair and the $[A(k), Q^\frac{1}{2}(k)]$ pair. These conditions, together with a persistent excitation condition (see Assumption 3 formulated later), ensure the stability of the adaptive Kalman filter presented in this paper.

3. THE ADAPTIVE KALMAN FILTER

In the adaptive Kalman filter, the state estimate $\hat{x}(k|k) \in \mathbb{R}^n$ and the parameter estimate $\hat{\theta}(k) \in \mathbb{R}^p$ are recursively updated at every time instant $k$. This algorithm involves also a few other recursively updated auxiliary variables: $P(k|k) \in \mathbb{R}^{n \times n}$, $Y(k) \in \mathbb{R}^{n \times p}$, $S(k) \in \mathbb{R}^{p \times p}$ and a forgetting factor $\lambda \in (0, 1)$.

At the initial time instant $k = 0$, the initial state $x(0)$ is assumed to be a Gaussian random vector

$$x(0) \sim \mathcal{N}(x_0, P_0).$$

Let $\theta_0 \in \mathbb{R}^p$ be the initial guess of $\theta$, $\lambda \in (0, 1)$ be a chosen forgetting factor, and $\omega$ be a chosen positive value for initializing $S(k)$, then the adaptive Kalman filter consists of the initialization step and the recursion steps described below. Each part of this algorithm separated by horizontal lines will be commented after the algorithm description.

Initialization

$$P(0|0) = P_0 \quad \quad Y(0) = 0 \quad \quad S(0) = \omega I_p$$

$$\hat{\theta}(0) = \theta_0 \quad \quad \hat{x}(0|0) = x_0$$

Recursions for $k = 1, 2, 3, \ldots$

$$P(k|k-1) = A(k)P(k-1|k-1)A^T(k) + Q(k)$$

$$\Sigma(k) = C(k)P(k|k-1)C^T(k) + R(k)$$

$$K(k) = P(k|k-1)C^T(k)\Sigma^{-1}(k)$$

$$P(k|k) = [I_n - K(k)C(k)]P(k|k-1)$$

$$Y(k) = [I_n - K(k)C(k)]A(k)Y(k-1) + [I_n - K(k)C(k)]\Phi(k)$$

$$\Omega(k) = C(k)A(k)Y(k-1) + C(k)\Phi(k)$$

$$\Lambda(k) = [\Lambda(k) + \Omega(k)S(k-1)\Omega^T(k)]^{-1}$$

$$\Gamma(k) = S(k-1)\Omega^T(k)\Lambda(k)$$

$$S(k) = \frac{1}{\lambda}S(k-1) - \frac{1}{\lambda}S(k-1)\Omega^T(k)\Lambda(k)S(k-1)$$

$$\hat{y}(k) = y(k) - C(k)[A(k)\hat{x}(k-1|k-1) + B(k)u(k) + \Phi(k)\hat{\theta}(k-1)]$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Gamma(k)\hat{y}(k)$$

$$\hat{x}(k|k) = A(k)\hat{x}(k-1|k-1) + B(k)u(k) + \Phi(k)\hat{\theta}(k-1) + K(k)\hat{y}(k)$$

$$\hat{y}(k) = y(k) - C(k)[A(k)\hat{x}(k-1|k-1) + B(k)u(k) + \Phi(k)\hat{\theta}(k-1)]$$

Recursions (5a)-(5d) compute the covariance matrix $P(k|k) \in \mathbb{R}^{n \times n}$ of the state estimate, the innovation covariance matrix $\Sigma(k) \in \mathbb{R}^{n \times n}$ and the state estimation gain matrix $K(k) \in \mathbb{R}^{n \times p}$. These formulas are identical to those of the classical Kalman filter. Inspired by the recursive least square (RLS) estimator with an exponential forgetting factor, recursions (5e)-(5i) compute the parameter estimate gain matrix $\Gamma(k) \in \mathbb{R}^{p \times p}$ through the auxiliary variables $Y(k) \in \mathbb{R}^{n \times p}$, $\Omega(k) \in \mathbb{R}^{m \times p}$, $S(k) \in \mathbb{R}^{p \times p}$. Equation (5j) computes the innovation $\hat{y}(k) \in \mathbb{R}^m$. Finally, recursions (5k)-(5l) compute the parameter estimate and the state estimate.

Part of equation (5l), namely,

$$\hat{x}(k|k) \sim A(k)\hat{x}(k-1|k-1) + B(k)u(k) + K(k)\hat{y}(k)$$

can be easily recognized as part of the classical Kalman filter, with the traditional prediction step and update step combined into a single step. The term $\Phi(k)\hat{\theta}(k-1)$ corresponds to the actuator fault term $\Phi(k)\theta$ in (1a), with $\hat{\theta}$ replaced by its estimate $\hat{\theta}(k-1)$. The extra term $\hat{y}(k)$ in the denominator of the prediction step is for the purpose of compensating the error caused by $\theta(k-1) = \hat{\theta}(k-1)$. This term is essential for the analysis of the properties of the adaptive Kalman filter in the following sections. It has also been introduced in the deterministic adaptive observer in (Guyader and Zhang, 2003), and its continuous time counterpart in (Zhang, 2002).

4. STABILITY OF THE ADAPTIVE KALMAN FILTER

Assumption 1. The matrices $A(k), B(k), C(k), \Phi(k), Q(k), R(k)$ and the input $u(k)$ are upper bounded, $Q(k)$ is symmetric positive semidefinite, and $R(k)$ is symmetric.
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