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Capital market equilibrium with moral hazard and flexible technology

John Quiggin[a,*], Robert G. Chambers[b]

[a] *Faculty of Business, Economics and Law, School of Economics, University of Queensland, St. Lucia 4072, Australia*
[b] *University of Maryland, College Park and University of Western Australia, Australia*

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Abstract

(Magill, M., Quinzii, M., 2002. Capital market equilibrium with moral hazard. Journal of Mathematical Economics 38, 149–190) showed that, in a stockmarket economy with private information, the moral hazard problem may be resolved provided that a spanning overlap condition is satisfed. This result depends on the assumption that the technology is given by a stochastic production function with a single scalar input. The object of the present paper is to extend the analysis of Magill and Quinzii to the case of multiple inputs. We show that their main result extends to this general case if and only if, for each firm, the number of linearly independent combinations of securities having payoffs correlated with, but not dependent on, the firms output is equal to the number of degrees of freedom in the firm's production technology.

Keywords: Capital market equilibrium; Moral hazard; Flexible technology

1. Inroduction

Magill and Quinzii (2002) showed that, in a stockmarket economy with private information, any state-contingent equilibrium may be generated as a financial market equilibrium provided that a spanning overlap condition is satisfed. Thus, with an appropriate specification of the security structure, the market can avoid the conflict between risk sharing and incentives that is typical of the moral hazard problem.

One notable feature of the analysis of Magill and Quinzii relates to the production technology, which is assumed to be characterized by a stochastic production function, with a single scalar

* Corresponding author. Tel.: +61 73346 9646; fax: +61 73365 7299.
E-mail address: j.quiggin@uq.edu.au (J. Quiggin).

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input. As observed by Holmstrom and Milgrom (1987), in a standard principal–agent problem, the greater the flexibility available to the agent, the more difficult the problem faced by the principal. For a highly flexible technology, Holmstrom and Milgrom show that the principal can do no better than to offer a payment schedule that is an affine function of output.

The object of the present paper is to extend the analysis of Magill and Quinzii to the case of a stochastic production function with multiple inputs. We show that their main result extends to this general case if and only if, for each firm, the number of linearly independent combinations of securities having payoffs correlated with, but not dependent on, the firm's output is equal to the number of degrees of freedom in the firm's production technology. The scalar-input stochastic production function technology examined by Magill and Quinzii is the case where the firm has a single degree of freedom. In the other polar case, where the firm has enough independent inputs to span the state space, the first-best can be achieved if and only if the state space is also spanned by 'outside' securities, independent of the firm's observed output.

2. Model

Magill and Quinzii (2002) consider a simple two-period one-good economy with production. There are two types of agents: entrepreneurs and investors. $\mathcal{I}_1 \neq \emptyset$ is the set of entrepreneurs, $\mathcal{I}_2 \neq \emptyset$ is the set of investors, and the set $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2$ of all agents is finite. Every agent $i \in \mathcal{I}$ has an initial wealth ω_0^i at date 0. An agent i who is an entrepreneur has the opportunity to create a productive venture by investing an amount of capital κ^i. We assume that there is a finite set $\mathcal{S}$ of states of nature describing the shocks to which firms can be subjected.

State-contingent output for agent i is given by a stochastic production function:

$$\mathbf{y}^i = \mathbf{F}^i(\kappa^i, \mathbf{e}^i) = (F_1^i(\kappa^i, \mathbf{e}^i), \ldots, F_S^i(\kappa^i, \mathbf{e}^i)),$$

where κ^i is a scalar capital input and $\mathbf{e}^i \in \Re_+^M$ is a vector of inputs that we refer to as effort, and each of the F_s^i is concave in effort. This is a generalization of the technology in Magill and Quinzii (2002), who restrict effort to be a scalar.[1]

We assume that J contracts are traded, contract j being characterized by a state-independent function $V^j : \Re^I \to \Re^I$ describing the way the payoff of contract j depends on the realized output of the I firms in the economy. The first I securities are assumed to denote equity shares in the I firms, with security i having payoff vector $\mathbf{y}^i$. For each i, the set of securities J can be partitioned into:

$$J = J^i \cup J_{-i}^i \cup J_{-i},$$

where J^i consists of securities for which the payoff depends exclusively on the output of firm i (including the equity of firm i and any derivative securities such as options), J_{-i}^i consists of securities for which the payoff depends on the output of firm i and the output of at least one other firm (such as index securities) and J_{-i} consists of securities for which the payoff does not depend on the output of firm i.

Let $\mathbf{y} = (\mathbf{y}^1, \ldots, \mathbf{y}^I)$ denote the random outputs of the firms and let $\mathbf{y}_s = (y_s^1, \ldots, y_s^I)$ denote the realized outputs in state s. The payoff of security j in state s is then $V^j(\mathbf{y}_s)$. We let $V^j(\mathbf{y})$ denote the vector $(V^j(\mathbf{y}_s))s \in \mathcal{S}$ and $\mathbf{V}(\mathbf{y}) = [V^1(\mathbf{y}), \ldots, V^J(\mathbf{y})]$ denote the matrix of payoffs of the J securities. The security price vector is $\mathbf{q} \in \Re^J$.

[1] The assumption that effort is a scalar is not made explicit until p. 171, but is implicit in the discussion throughout.