European option pricing under the Student's t noise with jumps

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Abstract

In this paper we present a new approach to price European options under the Student's t noise with jumps. Through the conditional delta hedging strategy and the minimal mean-square-error hedging, a closed-form solution of the European option value is obtained under the incomplete information case. In particular, we propose a Value-at-Risk-type procedure to estimate the volatility parameter such that the pricing error is in accord with the risk preferences of investors. In addition, the numerical results of us show that options are not priced in some cases in an incomplete information market.

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1. Introduction

It is generally accepted that fat tailed distributions occur commonly in finance. The distribution is fat tailed means that extreme price movements occur much more often than what a Gaussian model gives. In the fields of finance and econophysics, the tail distributions of the log-returns have been analyzed in a series of studies (e.g., see [1–19] for details) and have been found to be

\[ P (|r_t| > x) \sim x^{-\alpha} \quad \text{with} \quad \alpha \sim 3, \]
where $r_t$ denotes the logarithmic return over a time interval $\delta t$, and $\alpha$ is the tail or Pareto exponent. In particular, Stanley et al. [1,2,7,12] have presented a model in which volatility is caused by the trades of large institutions and have discovered that the above tail behavior is useful to guide and constrain the theory of the impact of large investors. Many other authors have shown that the distribution of logarithmic stock returns can be fitted extremely well by Student's $t$-distributions (e.g., see [3–6,8,9,14,15]). All those authors have advocated using a $t$-distribution with $\alpha$ degrees of freedom, typically such that $2.4 \leq \alpha < 5$. The Student's $t$-distribution is also used in finance and econophysics to price options. For examples, Bouchaud and Sornette [3], Borland et al. [9,14,15], McCauley et al. [20], and Cassidy et al. [21,22] considered the pricing of options with returns that are described by a Student's $t$-distribution and discovered that the integrals that determine the value of an option are infinite if the full domain of $(-\infty, +\infty)$ is used (e.g., see [3,20–22]). Therefore they suggested the truncated Student's $t$-distribution be used to price options, but they believed that truncation would lead to arbitrary prices for an option. Even if Cassidy et al. [21,22] argued that capping the value of the asset or that truncating the distribution of daily returns leads to prices for options while the returns of the underlying stocks are Student's $t$-variables, we think that it is still important to use the Student's $t$-distribution itself to price options. In addition, Moriconi [23] studied delta hedging while the distribution for stock returns followed a Student's $t$-distribution multiplied by a wide Gaussian and Lim et al. [24] studied the currency option pricing through using a generalized $t$-distribution. Another major feature of stock returns is that the sample paths of stock prices are discontinuous and that there exist jumps in stock prices, both in the stock markets and the foreign exchange markets (e.g., see [25–27]).

The above empirical findings bear an important implication for option pricing. The framework for option pricing was developed by Black and Scholes [28] and Merton [29] when the underlying asset follows a diffusion process. Closed-form solution for European options was derived by Merton [26] for a jump–diffusion process.

In this paper, on the basis of the points of view of Refs. [2–8,12,22,25–27,30–33], we will study the option pricing problem while the dynamics of stock price $S_t$ satisfies

$$d\ln S_t = \left(\mu - \frac{(\sigma \xi)^2}{2}\right) dt + \sigma d(W_t \xi) + \ln J_t dN_t,$$

where $\mu$ and $\sigma > 0$ are given constants, $W_t$ is a Wiener process, $N_t$ is a Poisson process with intensity $\lambda > 0$, $J_t$ is a positive stochastic process with independent and identical distribution, and a positive random variable $\xi$ whose density function is given by

$$f(x) = \frac{2b^a}{\Gamma(a)} x^{-a-1} e^{-\frac{b}{x^2}}, \quad x > 0,$$

where we assume that $a > 1$ and $b > 0$ are constants.

Let $\gamma = \frac{1}{x^2}$. We know that $\gamma$ follows a gamma distribution whose density function is given by

$$\psi(x) = \frac{b^a}{\Gamma(a)} x^{-a-1} e^{-bx}, \quad x > 0,$$

where we assume that $a > 1$ and $b > 0$ are constants.

Furthermore, if we assume that $W_t, J_t, N_t$ and $\xi$ are independent, from [8] we know that the density function of $W_t \xi$ is given by

$$p(t, x|a, b) = \frac{\Gamma(a + \frac{1}{2})}{\Gamma(a) \sqrt{2\pi bt}} \left(1 + \frac{x^2}{2bt}\right)^{-\left(a + \frac{1}{2}\right)},$$

which means that $W_t \xi$ follows a Student's $t$-distribution (e.g., see [8] for details).

Eq. (1.3) is a stock return density function model, which is proposed by econophysics researchers (e.g., see [8] for details). In [8], Gерig et al. gave some explanations to this model.

Noting that on the basis of the empirical findings the stock price Eq. (1.1) is proposed. In fact, we can generalize Eq. (1.1) through assuming that the density function of the random variable $\xi$ is a given function $\phi(x)$. For example, we can assume that $\xi$ follows a given gamma distribution, etc.

This paper is organized as follows. In Section 2, by using both conditional delta hedging strategy and minimal mean-square-error hedging, we will study the option pricing problem and deduce an option pricing formula while the evolution of stock price $S_t$ satisfies Eq. (1.2) as well as propose a new method to estimate the volatility parameter $\sigma$ such that the pricing error satisfies the risk preferences of investors. In Section 3 some examples are given to show that the risk preferences of traders and the exponent $\alpha$ have the important influences on option pricing and the “implied volatility smiles”. In Section 4, a conclusion is given.

### 2. European option pricing for a jump–diffusion economy with Student's $t$ noise

Let $(\Omega, F, (F_t)_{t \geq 0}, P)$ be a complete probability space carrying a standard Brownian motion $W = (W_t)_{t \in \mathbb{R}}$, a Poisson process $N = (N_t)_{t \geq 0}$ with intensity $\lambda > 0$, a positive stochastic process $J = (J_t)_{t \geq 0}$ with independent and identical
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