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Market Value Margin calculations under the Cost of Capital approach within a Bayesian chain ladder framework

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HIGHLIGHTS

- In the SII framework, insurance companies need to calculate the BEL and the MVM.
- The Cost-of-Capital approach is used.
- There exists an intricate circularity dependency between MVM and SCR.
- We consider a Bayesian log-normal chain ladder model.
- We give exact and accurate approximate analytic formulas for MVMs.

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ABSTRACT

In the Solvency II framework, insurance companies need to calculate the Best Estimate valuation of Liabilities (BEL) and the Market Value Margin (MVM) for non-hedgeable insurance-technical risks. The Cost-of-Capital approach defines the MVM as the present value of the current and future Solvency Capital Requirement (SCR) of the non-hedgeable risks to protect against adverse developments in the run-off of the insurance liabilities. However the SCR at time t itself depends on the increase in the MVM between t and $t + 1$. Hence there exists an intricate circularity dependency between both quantities. In this paper we present exact and accurate approximate analytic formulas for MVMs within a Bayesian log-normal chain ladder framework.

1. Introduction

Solvency II is creating a new approach to regulate capital requirements by quantifying risks and is giving incentives for companies to develop good risk management practices. It will be based on economic principles for the measurement of assets and liabilities and capital requirements will depend directly on this. In particular it introduces the market consistent economic (solvency) balance sheet and two points in time are considered to calculate the Solvency II Capital Requirement (SCR): the current balance sheet and the balance sheet at the end of the year. The main components of this balance sheet are the Market Value of Assets (MVA) and the Fair Value of Liabilities (FVL) consisting of the sum of two components: the Best Estimate valuation of Liabilities (BEL) and the Market Value Margin (MVM).

The BEL is the present value of expected future cash flows using best estimate assumptions with no explicit margins incorporated. However, for the non-hedgeable insurance risks, since there exists the risk that actual experience will be more adverse than expected, there needs to be an additional "risk margin" component added to the BEL, and the MVM. It can be interpreted as the cost of risk and uncertainty in the amount and timing of future payments needed to satisfy insurance liabilities.

For several decades, actuaries have used a variety of technical methods to consider the risk in the valuation of their insurance liabilities and risk margins have been implicitly or explicitly embedded in the assumptions or in the methods. Approaches for determining risk margins have been grouped into four families (see IAA position paper International Actuarial Association IAA, 2009):

- the quantile methods that use risk measures such as the Value-at-Risk (VaR), the Conditional Tail Expectation (CTE) and the Tail Value-at-Risk (TVaR) and define the risk margin as the difference of the risk measure of the discounted ultimate future payments and the BEL;
- the Cost-of-Capital (CoC) approach that defines the risk margin as the present value of the current and future SCRs for the non-hedgeable risks to protect against adverse developments in the run-off of the insurance liabilities;
- the discount related methods that define the risk margin as the difference of the discount expected cash flows using the risk-free interest rate minus a selected risk adjustment and the BEL

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(probability distortions to take into account risk aversion are alternatively used in Wüthrich et al., 2011);

- the methods that use explicit assumptions: the risk margin results from selecting prudent explicit parameters and simpler methodologies.

Over the recent years, the CoC approach has been preferred to estimate market-consistent risk margins for insurance contracts. One of the reason is that CoC approach is commonly used as a conceptual framework in both non-life and life insurance valuation applications. Solvency II prescribes that the MVM is calculated using the following 3 steps:

1. determine the expected SCRs for non-hedgeable risks until the run-off of the portfolio (remind that the SCR is defined as the amount of capital required to support the claims paid out and the increase in the sum of the BEL and MVM following a one-in-200-year event over the next year);
2. calculate the capital charge as the SCRs multiplied by the CoC charge and take the present value of the product;
3. take the sum of the present values for all years until the run-off to arrive at the MVM.

As can be read in this approach, estimating the SCRs for each future year in a theoretically correct way is far from straightforward because the SCRs appear to depend on the MVMs, and the MVMs depend on the SCRs. Hence there exists an intricate circularity dependency between both quantities. To mitigate the circularity issue, several simplifications have been suggested (from the more complex to the simpler):

- exclude the risk margin in the FVL within the calculations for the SCRs (see e.g. the Swiss Solvency Test);
- approximate the SCRs for the future years by using a 'proportional proxy': for example the current SCR is calculated, but the future SCRs are approximated by multiplying the current SCR by the ratio of the future BEL to the current BEL;
- approximate the current MVM by using a 'duration approach': the MVM is calculated as the current SCR multiplied by a modified duration of the insurance liabilities (see e.g. Ohlsson and Lauzeningks, 2009);
- approximate the current MVM by considering it as a percentage of the current BEL.

The problem of calculating the MVM in a theoretically correct way can only be solved by starting at the final time period and working backwards recursively. The problem is in general intractable without simplifications.

In Haslip (2010), Haslip considers the liabilities of a non-life insurance company and assumes that the outstanding claims reserve at the end of each year is subject to uncertainty through a scaling factor drawn from a log-normal distribution with mean 1 and several sets of coefficients of variation. He makes some comparisons with proxies derived from the proportional method, the duration method or by considering the MVM as a percentage of the current BEL. The results are: the duration method understates the risk margin, the proportional method provides a reasonable approximation but sensitive to the coefficients of variation of the log-normal distributions, the 'percentage of BEL' proxy is more conservative than the others.

In Bonnard and Margetts (2008) and Daya (2011), Bonnard, Daya and Margetts alternatively assume that the cumulative paid claims (for all the accident years) at the end of each year are equal to the cumulative paid claims at the beginning of the year multiplied by a scaling factor drawn from a log-normal distribution with deterministic but path dependent mean and volatility. They find that the first approximation (i.e. excluding the risk margin in the FVL within the calculations for the SCRs) gives higher results than the exact solution. The second and the third approximations (proportional and duration proxies) tend to underestimate the exact solution. The fourth approximation can be quite larger than the analytical solutions especially for the first development years.

In Salzmann and Wüthrich (2010), Salzmann and Wüthrich derive analytical formulas for the risk margin and compare different proxies but under the assumption that the capital requirements are defined through the standard error of the sum of the claims paid out and the increase in the BEL and the risk margin instead of the 99.5th percentile of the change as needed for the Solvency II approach.

In this paper, we consider a Bayesian log-normal chain ladder model for a non-life insurance company and extend the results of Bonnard, Daya and Margetts to the case (i) where recent information is immediately absorbed by the Bayesian model, (ii) where the cumulative paid claims of each accident year follow a multiplicative log-normal model and/or (iii) where the cash flows and the SCRs are discounted within the calculation of the BEL and the MVM. We give an exact analytical formula for the MVM of the liabilities of a specific accident year when reserves are not discounted. We propose accurate approximated analytic formulas for the MVM of the liabilities of all accident years (when reserves are discounted or not) by considering convex order techniques and approximations as used in Dhaene et al. (2002a,b) and Vanduffel et al. (2005, 2008).

The paper is organized as follows. In Section 2 we define the Bayesian log-normal chain ladder model for claims reserving and calculate the BEL depending on whether or not the cash flows are discounted. We then give general results concerning convex order approximations of sums of log-normal random variables in Section 3. In Sections 4 and 5 we provide respectively recursive analytical formulas for the calculation of the MVM when cash flows are not discounted or when they are subject to a constant discount rate. Finally, in Section 6 we provide a real data example that is based on liability insurance data. The proofs are deferred to Section 7.

2. Bayesian log-normal chain ladder model

We consider the liabilities of a non-life insurance company. The random variables $C_{i,j} > 0$ denote the cumulative payments of accident year $i \in \{1, \ldots, I\}$ after development year $j \in \{0, \ldots, J\}$, meaning that the incremental claims, $X_{i,j} = C_{i,j} - C_{i,j-1}$ for $j = 1, \ldots, J$, are paid in current accounting year $k = i + j$. We assume that all claims are settled after development J and that $I \geq J + 1$. We define the individual claims factor $F_{i,j} = C_{i,j}/C_{i,j-1}$ for $j = 1, \ldots, J$ and let $\xi_{i,j} = \log(F_{i,j})$. The first payment $C_{i,0}$ is the initial value of the process $\left(C_{i,j}\right)_{j=0,\ldots,J}$ and $F_{i,j}$ are the multiplicative changes.

The time unit corresponds to years, the current year is I and we write

$$\mathcal{T}_t = \sigma\{C_{i,0}, X_{i,j} : i + j \leq t, i = 1, \ldots, I, j = 1, \ldots, J\}$$

for $t = I, \ldots, I + J$.

We consider a Bayesian log-normal chain ladder model to complete the run-off trapezoid as it has been proposed by Merz and Wüthrich in Chapter 11 of their book (Merz and Wüthrich, 2013) to take into account parameter uncertainty. For the claims reserving problem, Bayesian methods are now well investigated (see e.g. Wüthrich, 2007, Bühlmann et al., 2009) and they provide an interesting approach for a successive information update in each accounting year. Log-normal chain ladder models have been introduced by Hertig (1985) and their Bayesian versions have been recently used by Merz and Wüthrich (2010) to model paid–incurred chain claims. We restrict ourselves to this model because it allows for explicit analytical formulas for the MVM of the liabilities of a specific accident year when reserves are not discounted.

We assume that

- given $\mathbf{\Phi} = (\Phi_0, \ldots, \Phi_{J-1})$ and $\sigma = (\sigma_0, \ldots, \sigma_{J-1})$, the $X_{i,j}$ are independent for different accident years i, and for $j = 0, \ldots,$