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Systems Engineering Procedia 3 (2012) 201 – 208

Procedia
Systems Engineering

Multi-Scale Approximate Entropy Analysis of Foreign Exchange Markets Efficiency

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Abstract

Market efficiency analysis is an important aspect in financial engineering.Based on weak-form efficient markets hypothesis (EMH), we characterize the market efficiency in foreign exchange (FX) markets by using the multi-scale approximate entropy (MApEn) to assess the randomness in FX markets. We split 17 daily FX rates from 1984 to 2011 into there periods by two global events, Southeast Asia currency crisis and American sub-prime crisis. The empirical results indicate that the developed FX markets is more efficient than emerging FX markets, and that the financial crisis promotes the market efficiency in FX markets significantly, especially in emerging markets, like China, Hong Kong, Korea and African market.

Keywords: Financial engineering; Entropy; Approximate entropy (ApEn); Multi-scale ApEn; Market efficiency;

1. Introduction

Suffering from the financial crisis started in 2008, the global monetary system has made a significant change, which spurs more attentions on the efficiency and complexity of foreign exchange markets [1-4]. Although the efficient-market hypothesis (EMH) has become controversial because of substantial and lasting inefficiencies are approved in empirical literatures, it is still an important starting point to study market efficiency. Following the innovative work of Fama [5], the financial market efficiency can be properly described by dynamic complexity or randomness, which is widely measured by entropy developed in econophysics (such as approximate entropy [3, 4, 6], cross-sample entropy [7, 8], information entropy [9-11] and Lempel-Ziv complexity [12]).

Entropy, a concept from statistical physics, is broadly used to quantify the disorder and uncertainty of complex dynamic systems. Considering the inherent non-linearity and noise of time series in physiology and medicine financial, Pincus [13] proposed the approximate entropy (ApEn) method based on information theory, and then applied to describe financial time series as a useful measurement of system stability, with rapid increases possibility on predicting remarkable changes in a financial variable [6]. In 2007, the ApEn method has been used to quantify the FX rate time series as a reliable estimator of the market efficiency in FX markets, and shows that the ApEn values for European and North American FX markets are generally larger than those for African and Asian ones except

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2211-3819 © 2011 Published by Elsevier Ltd. Selection and peer-review under responsibility of Desheng Dash Wu.
doi:10.1016/j.sepro.2011.10.030

Japan [3]. Eom et al [4] employed a symbolic transfer entropy and ApEn method to study information flows in FX rates and found a nearly positive relationship between the difference in the degree of asymmetry of information flows and the difference in the degree of market efficiency. For further characterizing the complexity, a multi-scale entropy (MSE) was introduced for complex time series and widely used in physiologic and biological time series [14, 15]. Moreover, Martina, et al [16] applied the MSE method to analyze the dynamical properties of crude oil price, the results of which shown the MSE approach can shed light on the structure of crude oil markets as well as on its link to macroeconomic conditions and socio-political extreme events.

Based on the empirical studies of ApEn and MSE methods, a multi-scale entropy (named as multi-scale ApEn or MApEn) approach is introduced in this paper. And our motivation is to characterize the market efficiency in FX markets and analyze the market reactions to global financial crisis, such as the Southeast Asia currency crisis and American sub-prime crisis, by the proposed MApEn measurement. The empirical result shows that the MApEn approach outperforms the ApEn method and overcomes the shortcoming of that the ApEn portrays the market efficiency merely on single scale. And we also find solid empirical evidences of that the financial crisis increases the market efficiency of FX markets considerably in general, especially of emerging markets.

The remaining is organized as follows. The next section presents the MApEn method which used to analysis the FX rate time series. Section 3 presents the EMH based on the MApEn method. Then, Section 4 gives the data statistics and the main empirical results. Finally, Section 5 concludes the paper.

2. Methodology

2.1. Approximate entropy (ApEn)

To analysis the short and noisy time series, Pincus [13] proposed the approximate entropy (ApEn) family of parameters, which have been wildly used to quantify the randomness inherent in physiology, medicine and finance time series. The algorithm for ApEn computation can be defined as follows:

Let x={x(1),x(2),...,x(N)} be the length-N time series. Fix input parameters m and r, where m is the embedding dimension, and r is the tolerance for accepting matches.

- Step 1 Create embedding vectors X(i), each is made up of m consecutive values of x,

$$\boldsymbol{X}(i)=\left[x(i),x(i+1),\cdots,x(i+m-1)\right],\qquad 1\le i\le N-m+1 . \tag{1}$$

- Step 2 The distance d[X(i), X(j)] between two vectors X(i) and X(j) is defined by

$$d\left[\boldsymbol{X}(i),\boldsymbol{X}(j)\right]=\max_{k=0,\cdots,m-1}\left\{\left|x(i+k)-x(j+k)\right|\right\} . \tag{2}$$

- Step 3 For each i, $1\le i\le N-m+1$, the probability that any X(j) is within r of X(i)

$$B_i^m(r)=\frac{\text{mumber of }1\le j\le N-m\text{ such that }d\left[\boldsymbol{X}(i),\boldsymbol{X}(j)\right]\le r}{N-m+1} , \tag{3}$$

where, the vector X(i) is so-called template, which is matched if a X(j) is within r of X(i). And then the average of $B_i^m(r)$ is defined by

$$\boldsymbol{B}^m(r)=\frac{1}{N-m+1}\sum_{i=1}^{N-m}\ln B_i^m(r) . \tag{4}$$

- Step 4 Increase the embedding dimension to m+1, and repeat the Step 1 to Step 3. We have