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Intraday volatility and scaling in high frequency foreign exchange markets

Lars Seemann *, Joseph L. McCauley, Gemunu H. Gunaratne

Department of Physics, University of Houston, Houston, TX 77204, United States

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ABSTRACT

Recent reports suggest that the stochastic process underlying financial time series is nonstationary with nonstationary increments. Therefore, time averaging techniques through sliding intervals are inappropriate and ensemble methods have been proposed. Using daily ensemble averages we analyze two different measures of intraday volatility, trading frequency and the mean square fluctuation of increments for the three most active FX markets; we find that both measures indicate that the underlying stochastic dynamics exhibits nonstationary increments. We show that the two volatility measures are equivalent. In each market we find three time intervals during the day where the mean square fluctuation of increments can be fit by power law scaling in time. The scaling indices in the intervals are different, but independent of the FX market under study. We also find that the fluctuations in return in these intervals lie on exponential distributions.

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1. Introduction

Analysis of financial time series has provided new insights about the underlying stochastic processes (Bouchaud & Potters, 2000; Dacorogna, Gençay, Müller, Olsen, & Pictet, 2001; Mantegna & Stanley, 2007; McCauley, 2009). Techniques from statistical physics have been adapted to analyze and model financial time series, to access risk, and price options. Early work by Osborne (1959, 1977) and Samuelson (1965) laid the foundation for the Black and Scholes option pricing model (Black & Scholes, 1973; Merton, 1973), which assumed that the stochastic dynamics of the underlying asset was a geometric Brownian motion. However, the hypothesis of Gaussian fluctuations disagrees with fluctuations seen in commodity markets as reported by Mandelbrot (1963, 1966).

Empirical studies conducted over the last two decades found that distributions of intraday fluctuations are non-Gaussian and contain fat tails (Cont, 2001; Dacorogna, Müller, Nagler, Olsen, & Pictet, 1993; Gopikrishnan, Plerou, Amaral, Meyer, & Stanley, 1999; Müller, Dacorogna, Olsen, Pictet, Schwarz, & Morgenegg, 1990; Olsen, Müller, Dacorogna, Pictet, Davè, & Guillaume, 1997; Schmitt, Schertzer, & Lovejoy, 1999; Xu & Gencay, 2003). For example, these distributions were found to follow a power law outside the Lévy stable domain (Gopikrishnan, Plerou, Liu, Amaral, Gabaix, & Stanley, 2000; Gopikrishnan et al., 1999; Plerou & Stanley, 2007). Furthermore, empirical analysis suggests that the distributions scale with the length of the time interval analyzed (Galluccio, Caldarelli, Marsili, & Zhang, 1997; Gopikrishnan et al., 2000; Vandewalle & Ausloos, 1998). Many of these analyses employed sliding interval methods, which implicitly assume that the underlying stochastic process X_t has stationary increments, i.e., the increments $\Delta X_\tau = \Delta X_{t,\tau} = X_{t+\tau} - X_t$ are independent of time t and are functions of τ only.

However, other reports have suggested that the increments are nonstationary, i.e., the increment $\Delta X_{t,\tau}$ is an explicit function of time. First, it was shown that the trading frequency is not uniform within a day. In fact, it was shown that the frequency varies by a factor of ~20 (Dacorogna et al., 2001; Müller et al., 1990; Zhou, 1996). Many authors proposed that financial market fluctuations are best analyzed in transaction time (Ane & Geman, 2000; Baviera, Pasquini, Serva, Vergni, & Vulpiani, 2001; Clark, 1973; Griffin & Oomen, 2008; Mandelbrot & Taylor, 1967; Oomen, 2006; Silva & Yakovenko, 2007). A second approach inferred that the volatility of the Euro–Dollar exchange rate (in real time) was not uniform and varied by a factor of around 3 within a day (Bassler, McCauley, & Gunaratne, 2007; Dacorogna et al., 2001; Müller et al., 1990; Zhou, 1996). Both approaches suggest that intraday increments are generally time dependent and one conclusion of the present work is that they are equivalent.

Bassler et al. (2007) demonstrated that there were several time intervals during which the Euro–Dollar exchange rate can be fit by power laws in time. Moreover, the scaling indices within these intervals were different. The second result of our work is that the scaling intervals and scaling indices are common for the three major currency exchange rates, EUR/USD, USD/JPY, and GBP/JPY. In fact, the volatility in these markets exhibits similar characteristics even outside the scaling intervals. We also ask whether price variations outside of the scaling intervals lie on exponential distributions as reported in Silva, Prange, and Yakovenko (2004); Bassler et al., 2007. We address this issue using low order absolute moments of the distributions.

* Corresponding author. Tel.: +1 832 232 1623; fax: +1 713 743 3589.
E-mail addresses: lseemann@uh.edu (L. Seemann), jmccauley@uh.edu (J.L. McCauley), gemunu@uh.edu (G.H. Gunaratne).

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Our studies are conducted on FX rates, which have the most active and liquid markets. The daily turnover in traditional FX market transactions in 2009 was approximately 3 trillion Dollars (BIS, 2007; IFSL, 2009). The market is open 24 hours on weekdays, i.e., Sunday 20:15 Greenwich Mean Time (GMT) till Friday 22:00 GMT. The global turnover can be accounted in three main geographical regions: Asia, Europe and North America (BIS, 2007; Galati & Heath, 2007). The UK accounts for 35.8% of exchange trading, while the US and Japan account for 13.9% and 6.7% respectively (IFSL, 2009). The three FX rates considered here were the most traded between 2001 and 2009 (BIS, 2007; IFSL, 2009). We restrict our analyses to trading days on which each recorded trade is reported with the bid and ask quote and approximate the spot price p as the average of the bid and ask price: $p = \frac{1}{2}(p_{bid} + p_{ask})$.

Following Osborne (1959), we analyze market fluctuations using the return $x(t;\tau) = log\frac{p(t+\tau)}{p(t)}$, where $p(t)$ represents the price of the commodity at time t. If the increments were stationary, the distribution of $x(t;\tau)$ would be independent of the starting time t, and would only depend on the time-lag τ. As we already mentioned, intra-day variations in trading frequency (Ane & Geman, 2000; Clark, 1973; Mandelbrot & Taylor, 1967) and volatility (Müller et al., 1990; Dacorogna et al., 2001; Zhou, 1996; Bassler et al., 2007) were used to argue that increments in FX rates were nonstationary.

We define volatility of returns as the root mean square fluctuation, see Eq. (2). If successive transactions are uncorrelated and the returns for each transaction are from the same unknown underlying distribution with finite variance σ_0 constant over time, then the standard deviation after M transactions is proportional to $\sqrt{M}\sigma_0$. Assuming that M transactions have been reported in a (short) time interval $[t, t+\tau]$, the standard deviation can be expressed as

$$\sigma(t;\tau) \propto \sqrt{\tau \nu_\tau(t)}\sigma_0, \tag{1}$$

where $\nu_\tau = \frac{M}{\tau}$ is the trading frequency. Thus, we suspect the volatility at a time t to be a function of the trading frequency. Here we define trading frequency as the number of recorded trades within a fixed time interval. Alternatively, we can define trading frequency by only considering trades within the time interval that change the price (tick time sampling) (Griffin & Oomen, 2008). We find however, that the choice of transaction time is the most appropriate for our analysis.

Fig. 1 illustrates the daily behavior of tick frequency and volatility according to Eq. (2). Both measures vary over the course of a day and exhibit similar complicated daily behavior. This means that the underlying stochastic process is not independent of the time of day.

Bassler et al. (2007) demostrated that the intra-day volatility for the EUR/USD exchange rate contained several intervals during which the fluctuations exhibited scaling; the scaling indices in these intervals were different. Here we wish to determine if the volatility in other FX markets are similarly time dependent, if there are scaling regions, and how the scaling intervals and indices of different markets are related. These studies are conducted using the mean-square-fluctuation of increments during the time interval $[t, t+\tau]$ over different trading days. Specifically,

$$\sigma^2(t;\tau) := \left\langle x^2(t;\tau) \right\rangle = \frac{1}{N}\sum_{k=1}^{N} x_k^2(t;\tau) \tag{2}$$

where N is the number of trading days and τ is chosen to be 10 min to eliminate correlations (Bassler et al., 2007). $x_k(t;\tau)$ represents the return in the interval $[t, t+\tau]$ on the k^{th} trading day. Note that applying an ensemble average is necessary because of the nonstationarity of the stochastic process. Methods based on sliding time averages of increments are not appropriate because the underlying dynamics exhibits nonstationary increments (McCauley, 2008). On the other hand, the use of ensemble averages is justified due to the approximate daily repetition of $\sigma(t)$ (Bassler et al., 2007).

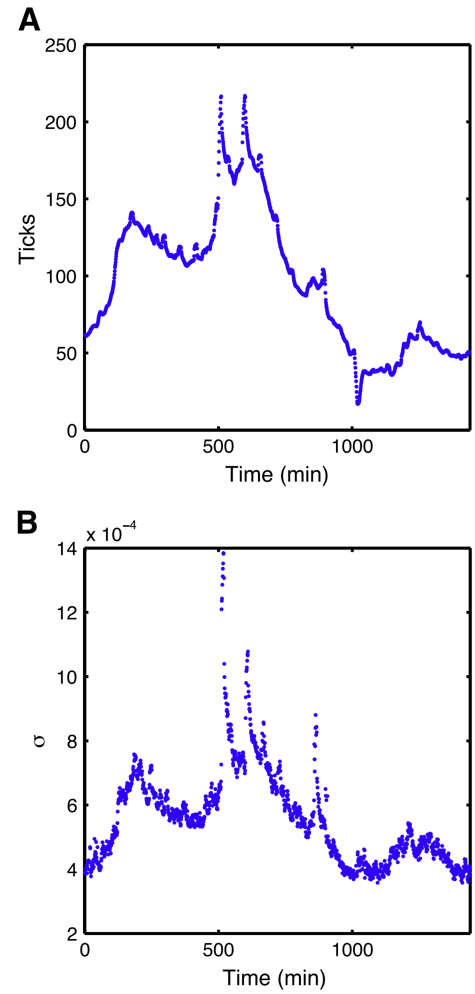

Fig. 1. A. The average number of ticks ν_τ of the EUR/USD exchange rate is plotted against time of day, with time lag $\tau = 10$ min. B. Volatility $\sigma(t;\tau)$ of the EUR/USD exchange rate is plotted against time of day, also with time lag $\tau = 10$ min to ensure that autocorrelations have decayed. The plots indicate that the underlying stochastic process is nonstationary and exhibits nonstationary increments, depending on starting time t. If the increments were stationary, σ would be flat. Times of high volatility (and high trading frequency) coincide with opening times of banks and financial markets in major financial centers. The peaks in plot B can be assigned to characteristic times during the trading day. Both measures exhibit similar daily behavior raising the question if they are related.

Next, consider the distribution $W(x,\tau;0)$ of fluctuations over a time lag τ starting from $t = 0$. In the scaling region, the scaling ansatz given by Bassler et al. (2007) asserts that

$$W(x,\tau;0) = \tau^{-H}\mathcal{F}(u), \tag{3}$$

where $\mathcal{F}$ is the scaling function of the scaling variable $u = \frac{x}{\tau^H}$ with the scaling index H. Note that t is set to zero at the beginning of each scaling interval. It was shown that the scaling function $\mathcal{F}$ of the EUR/USD rate within the scaling region between 9:00 AM and 12:00 AM New York time was close to bi-exponential. Here we compute the scaling functions for other scaling intervals and other FX markets.

Also note that we only have ~2000 ensembles in our study. This is insufficient to obtain accurate distribution functions. The method outlined in Bassler et al. (2007) first determines the scaling index H and subsequently uses the scaling ansatz, Eq. (3), and increments from multiple time intervals to compute $\mathcal{F}$.