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A class of asset pricing models governed by subordinate processes that signal economic shocks

Raj Jagannathan *

Department of Management Sciences, Tippie College of Business, The University of Iowa, Iowa City, IA 52242-1000, USA

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ABSTRACT

We consider a mean-reverting risk-neutral short rate process model with a vector of subordinated drift processes that accounts for the random effect of the arrival of new information. It is assumed that the market is efficient with no arbitrage opportunities. Closed form expressions for the price in *nominal* and in *real* terms of a discount bond are obtained. We define a risk-neutral exchange rate model with correlated subordinated drift and volatility processes that reflect the effect of the arrival of new information pertaining to the countries involved. The cases of complete and incomplete exchange markets with no arbitrage opportunities are considered.

1. Introduction

Evaluation of a discount (zero-coupon) bond price using a risk-neutral dynamic model of interest rate term structure in an arbitrage-free market is the subject of several papers. It is generally assumed that mean reversion is a desirable feature of the models considered, so that the short rate reaches a reasonable central value in the long run. In many of these models, the instantaneous variance is assumed to vary in proportion to square root of the short rate ensuring the nonnegative property of the short rate.

* Tel.: +1 319 335 0957; fax: +1 319 335 1956.
E-mail address: raj-jagannathan@uiowa.edu

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In evaluating a single factor term structure model, market data on zero-coupon yield curves are used in estimating the parameters of the model. Multi-factor models are generally used to improve the fit of the empirically observed term structure (yield curve) results. Some well-known single-factor arbitrage-free term structure models include Vasicek (1997) and the general equilibrium model by Cox et al. (1985). Merton (1976) suggests the use of jump processes to model the arrival of ''abnormal new information that has more than a marginal effect on stock prices''. In Duffie et al. (2000), a class of affine multi-factor asset pricing models with constant coefficients and jump diffusion processes is proposed, obtaining as special cases the jump diffusion models used in several other earlier papers.

It is known that fluctuations in the interest rate $r(t)$ and in the international exchange rate $S(t)$ of a dollar-denominated foreign currency move together in a predictable way and are influenced by common macroeconomic factors or economic shocks such as (i) changes in monetary and fiscal policies, (ii) introduction of new technology and (iii) price shocks as a result of the increase in the price of an input to the economy such as crude oil. We model the effects of these shocks as a collection of m (common) information-generating sources so that the total price change of an asset in any calendar time reflects the total effect of random arrival of *new* information. In the case of the international exchange rate $S(t)$, changes in monetary and fiscal policies of both countries under consideration are some of the influential factors.

In this paper we model the interest rate and exchange rate processes as Ito processes so that the SDE defined are governed by a common set of independent subordinated processes. A spot interest rate model is formulated with the following assumptions: (i) the mean-reverting property of a risk neutral term structure process is satisfied, and (ii) the drift and volatility terms are governed by correlated subordinated stochastic processes that reflect the accumulated effect of possible *prior arrivals of new information*. For each of the models considered, the computation required reduces to a simple numerical integration, for which several fast solution methods are available. Closed form expressions are obtained for yield and for prices in nominal terms and in real terms of a discount bond of arbitrary maturity.

Clark (1973) provides a clear justification for using a nonstationary subordinated process for modeling asset returns. The total price change of a traded asset in any calendartime reflects the accumulated effect of new arrival of information. On less eventful days, trading is slow and prices evolve slowly, while on more eventful days, prices evolve faster reflecting the increased speed of information arrival. In the context of the subordinated price model, the information arrival process is the natural candidate for the randomizing subordinator process and the price change process per information arrival is the parent process. Then asset returns become stationary if they are measured not in calendar time, but from the arrival to information arrival. In other words, the subordinated model stipulates that asset returns follow a stationary parent process in information-time with constant volatility even though asset returns follow a non-stationary subordinate process in calendar-time with stochastic volatility. Clark (1973) tests the SP model formulation by using futures data to show that a class of finite-variance distributions that are subordinate to normal processes fits the price data better than the stable family. Conley et al. (1997), Chang et al. (1998) and Piazessi (1998) use subordinated processes to signal arrival of new information.

In Section 2 of this paper, we define a process that defines the accumulated effect of arrival of new information over time. In Section 3, we consider a mean reverting risk-neutral term structure model of a short rate process. We assume the market to be efficient with no arbitrage opportunities and the drift and volatility terms of the defining stochastic differential equation to be correlated sums of subordinated stochastic processes. In Sections 4 and 5, closed form expressions for the price of a discount bond in nominal and in real terms are obtained. In Section 6, we consider exchange rate models and derive closed form expression for the price of foreign discount bonds and here we distinguish between complete/incomplete exchange markets. In Section 7, brief concluding remarks are given.

2. Accumulated effect of information arrival

Fluctuations in interest rates $r(t)$ are influenced by macroeconomic factors or economic shocks. We assume that we have m sources of information so that the total price change of an asset in any