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Physica A 364 (2006) 336–342
PHYSICA A
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# Correlation networks among currencies

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Available online 17 October 2005

**Abstract**

By analyzing the foreign exchange market data of various currencies, we derive a hierarchical taxonomy of currencies constructing minimal-spanning trees. Clustered structure of the currencies and the key currency in each cluster are found. The clusters match nicely with the geographical regions of corresponding countries in the world such as Asia or East Europe, the key currencies are generally given by major economic countries as expected.


*Keywords:* Econophysics; Foreign exchange market; Network

## 1. Introduction

A value of currency is expected to reflect the whole economic status of the country, and a foreign exchange rate is considered to be a measure of economic balance of the two countries. In the real world there are several economic blocks such as Asia, but it is not clarified whether such economic blocks affect the foreign exchange rate fluctuations or not. From the viewpoint of physics, the foreign exchange market is a typical open system having interactions with all information around the world including price changes of other markets. Also, the mean transaction intervals of foreign exchange markets are typically about 10 s, and it is not clear how the market correlates with the huge scale information of a whole country or the economic blocks. In order to empirically establish the relations between microscopic market fluctuations and macroscopic economic states, it is important to investigate the interaction of currency rates in the high precision data of foreign exchange markets.

The correlations among market prices have been analyzed intensively for stock prices by using minimal-spanning trees or self-organizing maps [1–5,14]. The interaction among stocks is expected to be caused by information flow, and direction of the information flow has been investigated from a cross-correlation function with a time shift [6–8]. Kullmann et al. and Kertesz et al. introduced a directed network among companies for the stocks [7,8]. We observe the interaction among foreign exchange markets using minimal-spanning tree.

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0378-4371/$ - see front matter 
doi:10.1016/j.physa.2005.08.079

We construct a currency minimal-spanning tree by defining correlation among foreign exchange rates as the distance. The minimal-spanning tree is a kind of currency map and is helpful for constructing a stable portfolio of the foreign exchange rates. We use correlation coefficient of daily difference of the logarithm rate in order to detect the topological arrangement of the currencies. The correlation coefficient is computed between all the possible pairs of rates in a given time period. We classify the currencies on the minimal-spanning tree according to the correlation coefficients, and find key currencies in each cluster. We analyze 26 currencies and 3 metals from January '99 up to December '03 provided by Exchange Rate Service [9].

## 2. Method of hierarchical taxonomy of currencies

We introduce a method of hierarchical taxonomy of currencies. We first define correlation function between a pair of foreign exchange rates in order to quantify synchronization between the currencies. We focus on a daily rate change $\mathrm{d}P_i(t)$ defined as

$$\mathrm{d}P_i(t) \equiv \log P_i(t + 1\,\mathrm{day}) - \log P_i(t), \tag{1}$$

where $P_i(t)$ is the rate $i$ at the time $t$. Using the rate change, correlation coefficient between a pair of the rates can be calculated by cross-correlation function as

$$C_{ij} = \frac{\langle \mathrm{d}P_i \cdot \mathrm{d}P_j\rangle - \langle \mathrm{d}P_i\rangle\langle \mathrm{d}P_j\rangle}{\sqrt{(\langle \mathrm{d}P_i^2\rangle - \langle \mathrm{d}P_i\rangle^2)(\langle \mathrm{d}P_j^2\rangle - \langle \mathrm{d}P_j\rangle^2)}}, \tag{2}$$

where $\langle \mathrm{d}P_i\rangle$ represents the statistical average of $\mathrm{d}P_i(t)$ for a given time. The correlation coefficient $C_{ij}$ has values ranging from $-1$ to 1.

We get $n \times n$ matrix of $C_{ij}$ by calculating the cross-correlation function for all combinations among the given rates when $n$ kind of foreign exchange rates are given. It is clear that the matrix has symmetry $C_{ij} = C_{ji}$ with $C_{ii} = 1$ from the definition of Eq. (2). We apply the correlation matrix to construct a currency minimal-spanning tree (MST), and can intuitively understand network among the foreign exchange rates using the MST. The MST forms taxonomy for a topological space of the $n$ rates. The MST is a tree having $n - 1$ edges that minimize the sum of the edge distances in a connected weighted graph of the $n$ rates. The edge distances satisfy the following three axioms of a Euclidean distance: (i) $d_{ij} = 0$ if and only if $i = j$, (ii) $d_{ij} = d_{ji}$, (iii) $d_{ij} \leqslant d_{ik} + d_{kj}$. Here, $d_{ij}$ expresses a distance for a pair of the rate $i$ and the rate $j$. We need Euclidean distances between the rates in order to construct the MST. However, the correlation coefficient $C_{ij}$ does not satisfy the axioms. We can convert the correlation coefficient by appropriate functions so that the axioms can be applied [1]. One of the appropriate functions is

$$d_{ij} = \sqrt{2(1 - C_{ij})}, \tag{3}$$

where $d_{ij}$ is a distance for a pair of the rate $i$ and the rate $j$.

We construct a MST for the $n$ rates using $n \times n$ matrix of $d_{ij}$. One of methods which construct MST is called Kruskal's algorithm [10,11]. The Kruskal's algorithm is a simple method consisting of the following steps: in the first step we choose a pair of rates with nearest distance and connect with a line proportional to the distance. In the second step we also connect a pair with the second nearest distance. In the third step we also connect the nearest pair that is not connected by the same tree. We repeat the third step until all the given rates are connected in one tree. Finally, we achieve a connected graph without cycles. The graph is a MST linking the $n$ rates.

We introduce maximal distance $\widehat{d}_{ij}$ between two successive rates encountered when moving form the starting rate $i$ to the ending rate $j$ over the shortest part of the MST connecting the two rates. For example, the distance $\widehat{d}_{ad}$ is $d_{bc}$ when the MST is given as

$$a - b \;\; — c - d,$$

where $d_{bc} \geqslant \max\{d_{ab}, d_{cd}\}$. The distance $\widehat{d}_{ij}$ satisfies axioms of Euclidean distance and a following ultrametric inequality with a condition stronger than the axiom (iii) $\hat{d}_{ij} \leqslant \hat{d}_{ik} + \hat{d}_{kj}$ [12],

$$\widehat{d}_{ij} \leqslant \max\{\widehat{d}_{ik}, \widehat{d}_{kj}\}. \tag{4}$$