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Physica A 353 (2005) 403–412
PHYSICA A
www.elsevier.com/locate/physa

Time and foreign exchange markets

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Received 29 July 2004; received in revised form 23 December 2004
Available online 14 March 2005

Abstract

The definition of time is still an open question when one deals with high-frequency time series. If time is simply the calendar time, prices can be modeled as continuous random processes and values resulting from transactions or given quotes are discrete samples of this underlying dynamics. On the contrary, if one takes the business time point of view, price dynamics is a discrete random process, and time is simply the ordering according to which prices are quoted in the market. In this paper, we suggest that the business time approach is perhaps a better way of modeling price dynamics than calendar time. This conclusion comes from testing probability densities and conditional variances predicted by the two models against the experimental ones. The data set we use contains the DEM/USD exchange quotes provided to us by Olsen & Associates during a period of one year from January to December 1998. In this period, 1,620,843 quotes entries in the EFX system were recorded.

PACS: 89.65.Gh; 05.40.Fb

Keywords: Forex markets; Time; Lags; High frequency

1. Introduction

In the high-frequency arena there are two mainstreams about modeling the stochastic properties of quotes. The first approach is to consider quotations as

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2005.01.049

sampled values of an underlying continuous-time random process [1,2]. Sampling is itself a random operation, thus introducing a twofold uncertainty in the price determination [3,4]. In this framework, time in the model flows continuously, and is called **calendar time**.

In the second approach, quoted prices are modeled through a discrete-time stochastic process [5]; in this setting, time is just the natural total order relation among quotations, and it is isomorphic with the set of non-negative integers (time being 0, the time associated to the first considered quotation). This is the **business time** approach, and randomness only enters in the determination of prices. It should be pointed out, however, that the waiting times between two quotes are also random quantities, but they are assumed to not contribute to the price determination process.

Whether a calendar-time or a business-time framework should be adopted in modeling the stochastic nature of financial quotes has been a longly debated issue by the finance research community, and it clearly depends on many factors, like, for example, (a) adherence to the physical behavior of reported prices, (b) usefulness in terms of a theory to be developed, and (c) last but not least, a matter of taste. See, for example [6–8].

In this paper, we suggest that business time is perhaps a better tool for modeling the asset dynamics than calendar time. In order to support our claim, we consider (1) returns corresponding to a given calendar time lag and any business time lag, (2) returns corresponding to the same calendar time lag but having a fixed business time lag. We find out that their statistical properties are different consistently with the business hypothesis and inconsistently with the calendar one. In practice, we estimate some variances and some probability densities whose behavior is different in the two scenarios.

The data set we use contains the DEM/USD exchange quotes taken from Reuters' EFX pages (the data set having been supplied by Olsen & Associates) during a period of 1 year from January to December 1998. In this period, 1,620,843 quotes entries in the EFX system were recorded. The data set provides a continuously updated sequence of bid and asks exchange quotation pairs from individual institutions whose names and locations are also recorded. The reason for using FX data is that this market is not subject to any working time restriction; in fact, it is open 24 h a day, seven days a week. This is in contrast to stock markets, where artificial time regulation would have made it more difficult, if not impossible, to find out the results outlined in this paper.

2. Business time vs. calendar time

2.1. Calendar time

In the calendar time framework, prices are modeled as continuous-time random processes. Clearly, market quotes are not defined for every $t \in \mathbb{R}$, but only at discrete intervals, whose extensions in time are called *calendar lags* (usually ranging from 2 s to several minutes, sometimes hours). Nevertheless, according to the calendar time