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Genetic learning as an explanation of stylized facts of foreign exchange markets

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Abstract

This paper revisits the Kareken–Wallace model of exchange rate formation in a two-country overlapping generations world. Following the seminal paper by Arifovic [Journal of Political Economy 104 (1996) 510] we investigate a dynamic version of the model in which agents' decision rules are updated using genetic algorithms. Our main interest is in whether the equilibrium dynamics resulting from this learning process helps to explain the main stylized facts of free-floating exchange rates (unit roots in levels together with fat tails in returns and volatility clustering). Our time series analysis of simulated data indicates that for particular parameterizations, the characteristics of the exchange rate dynamics are, in fact, very similar to those of empirical data. The similarity appears to be quite insensitive with respect to some of the ingredients of the genetic algorithm (i.e. utility-based versus rank-based or tournament selection, binary or real coding). However, appearance or not of realistic time series characteristics depends crucially on the mutation probability (which should be low) and the number of agents (not more than about 1000). With a larger population, this collective learning dynamics looses its realistic appearance and instead exhibits regular periodic oscillations of the agents' choice variables.

JEL classification: D83; D 84; F31

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1. Introduction

Foreign exchange markets as well as other financial markets are characterized by a number of striking ubiquitous time series features. Most prominently, (log) exchange rates seem to be non-stationary while their first differences are stationary. More precisely, unit root tests are typically unable to reject the null hypothesis of a first-order autoregressive process with a coefficient equal to unity. This finding squares with the well-known result of Meese and Rogoff (1983) that random walk forecasts produce a lower mean-squared error in out-of-sample prediction than reduced-form structural models of macroeconomic fundamentals. It has been argued that these findings can be explained by *speculative efficiency* of foreign exchange markets, which simply means one interprets the foreign exchange market as an informationally efficient market in the sense of the Efficient Market Hypothesis (cf. Bilson, 1981).

While from this perspective the unit root property may not be viewed as a conundrum, other well-known features have defied straightforward explanations until recently. The most pervasive ones are the fat-tail property of relative price changes and the clustering of volatility in these time series. Traces of these features are easily recognizable in all records of high-frequency data (probably up to weekly frequency) of foreign exchange markets (to our knowledge, without any known exception). The fat-tail property implies that the unconditional distribution of daily returns (as well as those of higher and somewhat lower frequency) has more probability mass in the tails and the center than the standard Normal distribution. This also means that extreme changes occur more often than would be expected under the assumption of Normality of relative daily price changes. Volatility clustering means that periods of quiescence and turbulence tend to cluster together. Hence, the volatility (conditional variance) of exchange rate changes is not homogeneous over time, but is itself subject to temporal variation.

Explanations of these stylized facts have been elusive until very recently. Perhaps, the silence of economic theory on this issue is not too surprising given that the above regularities are features of time series as *a whole* and, hence, could only be explained by dynamic models of the evolution of the trading process in the pertinent market. From the viewpoint of informational efficiency, the characteristics of returns would, of course, have to be explained by similar characteristics of the *news arrival process*, but due to the unobservability of the later, this hypothesis can hardly be subjected to econometric scrutiny. As an alternative, some authors have recently argued that fat tails and clustered volatility can be obtained as a result of interactions of heterogeneous economic agents. Examples of this emergent literature include Lux and Marchesi (1999, 2000), Chen et al. (2001), Kirman and Teyssière (2002), Gaunersdorfer and Hommes (2000), Chiarella and He (2001), Iori (2002) and Bornholdt (2001). Lux and Marchesi, Gaunersdorfer and Hommes, and Chiarella and He have models of fundamentalist—chartist interaction in financial markets which give rise to realistic behavior of the resulting time series (in terms of the above stylized facts). In Lux and Marchesi and Gaunersdorfer and Hommes, the authors try to provide some hints of general mechanisms that could generate these time series properties irrespective of the details of their exemplary models. In the former case, it is a critical behavior of the dynamics in the vicinity of a continuum of equilibria with an indeterminate composition of the population in terms of strategies pursued by individuals. Gaunersdorfer and Hommes get