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Analysis of Fokker–Planck approach for foreign exchange market statistics study

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Abstract

In a well-known work (Phys. Rev. Lett. 84 (2000) 5224) it was shown that behaviour of returns for foreign exchange markets in different time scales can be described in terms of Fokker–Planck equation, with Kramers–Moyal coefficients being estimated from the empirical data. In the current paper the authors provide analytical solution for stationary Fokker–Planck equation, which allows explanation of non Gaussian tails of the distribution function. It is also shown that while approximating empirical data one needs to observe some limitations for correct results obtaining.

Keywords: Fokker–Planck; Foreign exchange market

1. Model

Stochastic methods are powerful means for foreign exchange markets analysis. Typically the differences in exchange rates with fixed time delays are investigated. These data can be regarded as a stochastic process. It was shown in Ref. [1] that the empiric data of U.S. dollar–German mark exchange rates upon different delay times

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can be regarded as a stochastic Marcovian process. Further it was shown in the same paper that Kramers–Moyal coefficients can be estimated from the empirical data and finally, an explicit Fokker–Planck equation, which models the empirical probability distributions for reverse logarithmic time scale can be obtained:

$$\frac{\mathrm{d}}{\mathrm{d}\tau} p(\Delta x,\tau) = \left[-\frac{\partial}{\partial \Delta x} D^{(1)}(\Delta x,\tau) + \frac{\partial^2}{\partial \Delta x^2} D^{(2)}(\Delta x,\tau) \right] p(\Delta x,\tau) \,. \tag{1}$$

Here $\tau = \ln(40\,960/\Delta t)$ where Δt is time difference and Δx is the difference in exchange rate, $\Delta x \in (-\infty, +\infty)$. Coefficients $D^{(1)}$ and $D^{(2)}$ are Kramers–Moyal coefficients.

In these co-ordinates the point $\tau = +\infty$ corresponds to $\Delta t = 0$. Initial conditions are set for $\tau = 0$, which corresponds to $\Delta t = 40\,960\,\mathrm{s}$.

2. Analytical solution

In paper [1] numerical investigation of the obtained Fokker–Planck equation (1) was performed, but an analytical study of the equation properties seems to be quite important, as it could explain some of the properties of the numerical solution, performed in Ref. [1].

It seems to be quite interesting to investigate the behaviour of the distribution function in the vicinity of $\tau = +\infty$; it is also important to understand if the equation correctly describes the properties of the distribution function in the extreme case. To do that one needs to find the stationary solution of equation (1). In order to do that we will transform the equation into the conservative form:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\tau} p(\Delta x,\tau) = \frac{\partial}{\partial \Delta x} \Bigg[& \exp\left(\int \frac{D^{(1)}(\Delta x,\tau)}{D^{(2)}(\Delta x,\tau)} \, \mathrm{d}\Delta x \right) \\ & \times \frac{\partial}{\partial \Delta x} \left\{ D^{(2)}(\Delta x,\tau) \exp\left(-\int \frac{D^{(1)}(\Delta x,\tau)}{D^{(2)}(\Delta x,\tau)} \, \mathrm{d}\Delta x \right) p(\Delta x,\tau) \right\} \Bigg] \,. \end{aligned} \tag{2}$$

Thus stationary solution of the equation (2) can be given by

$$p(\Delta x) = \lim_{\tau \to +\infty} \frac{C \exp\left(\int \frac{D^{(1)}(\Delta x,\tau)}{D^{(2)}(\Delta x,\tau)} \, \mathrm{d}\Delta x \right)}{D^{(2)}(\Delta x,\tau)} \,. \tag{3}$$

In this formula C is a normalization constant, defined so, that $\int_{-\infty}^{+\infty} p(\Delta x) \mathrm{d}\Delta x = 1$.

In paper [1] the empirical data were used to approximate coefficients $D^{(1)}$ and $D^{(2)}$. $D^{(1)}$ was linearly approximated as a function of Δx and $D^{(2)}$ was approximated as a quadratic function of Δx summed with an $\exp(-\tau/2)$. Thus these coefficients have the following shape:

$$D^{(1)}(\Delta x,\tau) = -a_1 \Delta x \tag{4}$$

$$D^{(2)}(\Delta x,\tau) = a_2 \exp(-\tau/2) + b_2(\Delta x + c_2)^2 \,. \tag{5}$$

All the coefficients are positive.