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# Inverse statistics in the foreign exchange market

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**Abstract**

We investigate intra-day foreign exchange (FX) time series using the inverse statistic analysis developed by Simonsen et al. (Eur. Phys. J. 27 (2002) 583) and Jensen et al. (Physica A 324 (2003) 338). Specifically, we study the time-averaged distributions of waiting times needed to obtain a certain increase (decrease) $\rho$ in the price of an investment. The analysis is performed for the Deutsch Mark (DM) against the US$ for the full year of 1998, but similar results are obtained for the Japanese Yen against the US$. With high statistical significance, the presence of "resonance peaks" in the waiting time distributions is established. Such peaks are a consequence of the trading habits of the market participants as they are not present in the corresponding tick (business) waiting time distributions. Furthermore, a new *stylized fact*, is observed for the (normalized) waiting time distribution in the form of a power law Pdf. This result is achieved by rescaling of the physical waiting time by the corresponding tick time thereby partially removing scale-dependent features of the market activity.


*Keywords:* Inverse statistics; Econophysics; Interdisciplinary physics

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## 1. Introduction

Per Bak was a great scientist and a fantastic source of inspiration for many of us over many years. Through numerous lively and exciting discussions with him, one

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always felt that a project or a calculation was brought back on track again by his clever comments and suggestions. He applied his ingenious idea of "self-organized criticality" to many different systems ranging from sand piles, earthquakes to the brain and even finance. As he said: "It's all the same", meaning that in the end the paradigm of the sand pile model would after all describe the behavior of the particular system he considered. The idea of applying inverse statistics to turbulence data was the subject of the discussion between Per Bak and one of us (MHJ) several times. He liked the idea, and as such, we are quite sure that he would have liked our application of inverse statistics to financial data. This in particular applies to the scale invariant power-law scaling that is being observed for the normalized waiting time distribution. It is therefore our pleasure to dedicate this paper to his memory.

With the financial industry becoming fully computerized, the amount of recorded data, from daily close all the way down to tick-to-tick level, has exploded. Nowadays, such tick-to-tick high-frequency data are readily available for practitioners and researchers alike. In general, such high-frequency data are irregularly spaced in (physical) time, since an actual trade is a negotiation between sellers and buyers through a bid and ask process highly influenced by the irregular flow of information reaching the market. Hence, in order to apply the classic return approach to such data, the asset price has to be re-sampled equidistantly in physical time. This has been suggested in the seminal paper on high-frequency foreign exchange (FX) data analysis published by the Olsen & Associates Research Institute [3], but in many ways such a re-sampling violates the true dynamics of the market. Consequently, there has been an increasing interest over the past decade in studying variations in the market over a *variable* time span opposed to that of a *fixed* time span as for the return distribution [1,2,4]. One such approach is to consider drawdowns/ups, where an increasing or decreasing trend is followed to the end [6,7]. Recently, the present authors MHJ, AJ and IS introduced another such time varying approach—the *inverse statistics approach* [1,2]. At the heart of this technique, lies the waiting time needed to cross a pre-described return barrier. [1] The distribution of these waiting times, also termed investment horizons, characterize the inverse statistics [5] and has successfully been applied to daily close stock index data [1,2].

The purpose of the present paper is to follow up on these studies and investigate the corresponding statistical distributions for the FX market using high-frequency data. In particular, this work focuses on the exchange rate for the full year of 1998 between the two major currencies of the world, namely the US$ and the Deutsch Mark (DM), the latter in 2000 replaced by the Euro.

## 2. Formalism

Before we present the results of our analysis, we will set the stage by recapitulating a few important definitions and properties of inverse statistics. A more detailed

[1] One may also consider the completion process of a trade as the crossing of the bid and ask random walks.